

Implications of Causal-Realist Preference Theory on Expected Utility Theory

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Abstract

We examine Causal-Realist preference theory with a focus on the completeness axiom and discuss its implications on Expected Utility Theory (EUT). We show that one can represent preferences by a utility function in the realm considered in standard Causal-Realist analysis. We present several interior critiques of EUT, and conclude that EUT is neither apodictically nor empirically valid and hence reject it. We then consider what a theory of decision-making under conditions of risk would have to look like if it is derived from a priori true statements concerning human action.

Introduction

At the heart of economic theory, is the concept of value or utility, as it is often called. All of economic action is founded upon the analysis of actor's choices as they move from a state of affairs that they value less to one that they value more. To put it in terms that all Causal-Realists are familiar with, people act. Economics started out as a profession geared to studying how people or countries could maximize their wealth. However, the marginalist revolutionaries' insight that everything is valued subjectively led to the epiphany that there is nothing inherently unique about economizing money. Its inherent desirability simply lies in its effectiveness at being able to move someone from one state of the world to another.

The marginalist revolutionaries' successors have analyzed how actors value different things in different situations and hence how people act in those different situations. Becker in his influential work "Crime and Punishment: An Economic Approach"(1968) provides us with a good example of how economists have applied this concept of value. However, a far more important example and one more immediately relevant to the topic of this paper is the work of Menger and those who followed in his steps in developing the theory of time preference. This theory explains how people react

to the physical reality that all action takes place in time, and for many actions, a noteworthy period of time occurs between the costs incurred in the action and the desired result. Because of the sheer ubiquity of this phenomena, its analysis has aided in explaining many different phenomena, especially topics such as capital theory and the boom-bust cycle (French 2011). The temporality of all human action along with the finiteness of human actors gives rise to another universal reality of human action—risk, or if you prefer, uncertainty. Because the future is uncertain and all action occurs in time the outcome of any action is inherently not perfectly knowable. There is always the potential that one will not achieve the desired end by use of your chosen means, i.e. uncertainty applies to all human action.

Different economists have also tried to develop a theory of action under risk or uncertainty. However, unlike time preference theory, they have not been able to develop a theory that breeds any conviction especially when one examines what the theory predicts about the world in which we live. The standard theory, Expected Utility Theory, was developed by John Von Neumann and Oskar Morgenstern in their classic work *Theory of Games and Economic Behavior* (1953). However, since 1953 the evidence that has mounted against Expected Utility Theory has become too damning to ignore, and many of the alternative models that have been developed, whose number runs well into the double figures (Starmer 2000, 332), are either too psychological or too nebulous to be able to replace it. Their approach was axiomatic, and one of the axioms that Von Neumann and Morgenstern used in developing their theory is the axiom of complete preferences. This axiom has never meshed with Causal-Realist utility theory, and

relaxing this axiom provides a ray of light into the otherwise murky waters of behavior under conditions of risk. This paper will examine the Causal-Realist perspective on the structure of preferences and then in that light explain and analyze Expected Utility Theory, and then examine what one can say about people's behavior under conditions of risk. We will use the standard definition of risk, simply, actions where the future is undetermined from the point of view of the actor, but the numeric probabilities of the different outcomes are known.

This paper will limit itself to situations where the probabilities of the outcomes are known. Conditions where they are not known, often referred to as Knightian uncertainty, is another extremely interesting topic and fruitful avenue of further research. However, attempting to include that analysis in this paper as well would dilute the work where probabilities are known and this paper would not do either problem justice. Furthermore, this paper will not delve into the problems with the methodology used in the literature regarding Expected Utility Theory and instead focus on the economic conclusions in that literature.¹

Introduction to Expected Utility Theory

As mentioned above, John von Neumann and Oskar Morgenstern first developed Expected Utility Theory in their book *Theory of Games of Economic Behavior*. Their contribution resided in deriving a very tractable model from a small number of, at least initially, intuitively appealing axioms. Furthermore, they showed that one could represent

¹ If one is interested in a more general critique of the methodology used, which may also invalidate the conclusions independently, there are numerous works on the subject. One can start with Rothbard's *Man, Economy and State with Power and Market* (2004).

ordinal preferences in terms of a utility function as long as their axioms hold. As such, their work echoes that of many Causal-Realists in that they attempt to start with universally true statements about human action and then develop a set of more specific statements from it. Furthermore, as noted above, their use of mathematical proof implies that if those axioms are true, then their conclusions must be as well. However, the tenuousness of their axioms raises significant questions about their predictions, and, as will be shown later, the absurdity of their predictions implies that at least one of their original axioms is false at least some of the time.

We will follow the approach of Starmer in his article “Development in Non-Expected Utility Theory” (2000) in laying out the four axioms required to deduce the expected-utility hypothesis. The first, and least controversial, of these axioms is the requirement that preferences be transitive. That is if A is weakly preferred to B , and B is weakly preferred to C , then A is weakly preferred to C . To put it in the notation normally used in the Neoclassical literature: *If $A \succeq B$, and $B \succeq C$, then $A \succeq C$.* We can immediately accept this axioms as valid for three reasons. 1) It does not make any sense for someone to have intransitive preferences. If someone chooses a loaf of bread over a two dollars, and two dollars over stick of butter, it would be irrational for the person to, at the same time, choose a stick of butter over a loaf of bread. Furthermore, if people had these intransitive preferences then another person could create a system of exchanges that when the person would agree to but leave him worse off. 2) Causal-Realist economists accept this axiom. For example, Rothbard (1997, 216-217) points out how people have rational behavior implies consistent (i.e. transitive) preferences. Mises (1998, 103) makes

a similar statement in his magnum opus *Human Action*, stating that rationality requires transitive preferences. 3) Neoclassical economists also accept this axiom without reservation (Mas-Colell, Whinston, and Green 1995, 6-8).

The next three axioms are substantially more controversial, and so we will delay our analysis of them until later in the paper, and will just explain them without comment here. The first of these is the assumption of complete preferences. For any two states of the world, the person prefers the first one, prefers the second one, or is indifferent between them, that is for two states of the world A and B , either $A \succeq B$, $B \succeq A$, or $A \sim B$.

The third axiom is the axiom of continuous preferences. In this context, it means something slightly different than in a more general context because one uses probability in the definition instead of assuming preferences for parts of goods. If someone has preferences over three states: A , B & C where A is preferred to B , and B to C , then there is a combination with some probability of A and some probability of C where the person is indifferent between that combination and B , that is there exists a probability p , such that *if $A \succeq B \succeq C$, then $pA + (1 - p)B \sim C$.*

The third axiom used, and the most controversial, is the axiom of independence of irrelevant alternatives. It says that if you have three states A , B & C where A is weakly preferred to B , then some probability of A plus some probability C is weakly preferred to same probability of B plus the same probability of C . That is for some probability p , *if $A \succeq B$, then $pA + (1 - p)C \succeq pB + (1 - p)C$.*

As mentioned previously, John Von Neumann and Oskar Morgenstern showed that these axioms implied the expected utility hypothesis. That hypothesis is simply the

idea that people's preferences in the face of risk can be modeled with the following formula. Let L' and L'' represent two lotteries, i.e. probabilistic combinations of goods.² Let u_1, u_2, \dots, u_N represent the utility derived from the various goods. Let p_1', p_2', \dots, p_N' represent the probabilities of goods 1, 2, ..., N associated with L' , and $p_1'', p_2'', \dots, p_N''$ represent the probabilities of goods 1, 2, ..., N associated with L'' . Then we have

$$L' \succeq L'' \text{ if and only if } \sum_{n=1}^N u_n p_n' \geq \sum_{n=1}^N u_n p_n''.$$

This implies that preferences under conditions of risk can be represented by a monotonic utility function that is unique to a linear transformation.³ All the expected utility hypothesis is saying is that the utility derived from a given probabilistic combination of outcomes is simply the utility derived from each of the outcomes weighted by the probability of that outcome. To put it in mathematical terms, the utility derived from a lottery is precisely the same as the expected utility of that lottery, hence the theory's name. The utility derived is not normally the utility of the expected or average outcome. For example, the utility derived from a bet of \$10.00 versus \$0.00 at even odds, is .5 times the utility of \$10.00 plus .5 times the utility of \$0.00. It is usually not the utility of \$5.00.

Causal Realist Utility Theory

Now, let us consider what Causal-Realists would argue we do actually know about people's preferences. The literature is rather sparse on what form preferences may

² We use the word "good" to simply refer to something the person values and make no stipulation regarding what it actually is.

³ For a proof of these two results see Mas-Colell, Whinston, and Green (1995, 173-178).

take aside from the time preference question, which does not directly concern us here. However, they do have a utility theory that is worth examining. The first question in any utility theory is simply what does it mean to prefer something? There are two operationally equivalent definitions given in the economic literature. The first of these is simply that to prefer something means that we value it or derive satisfaction from it. The other says that if we prefer one thing to another we will chose the thing that we prefer. Both of these definitions are used in both the Neoclassical and the Causal-Realist literature. For example, Mises (1998, 13), in his seminal work *Human Action* uses the two definitions synonymously in his definition of action saying, “[Action] means nothing else than man’s faculty to choose between different states of affairs, to prefer one, to set aside the other, and to behave according to the decision made in aiming at the chosen state and forsaking the other.” Israel Kirzner in his work *The Economic Point of View* (1960, 102), says, “When forced to choose between two alternatives, the individual exercises his preference in a way that remains essentially the same regardless whether alternatives represent ‘positive’ or ‘absolute’ utilities.”

The point Kirzner is discussing here is quite important. What is the distinction between “absolute” and “positive” utilities? Whenever you choose one thing over another, you are comparing two different values. The general structure of these comparisons can take two forms: 1) You compare all goods relative to some common denominator or at least you can reduce your preferences down to a comparison with a common denominator. 2) You just have a system of pairwise comparisons, perhaps not complete but complete enough to be able to derive a pairwise comparison between most

combinations of most goods. The key distinction between these two approaches is in the first you can talk about valuing something for its own sake, and hence the first allows you to talk about the utility derived from consuming that good. The second however reduces preference to simply being, “I would choose *A* over *B*, if I was given the choice.” One cannot value something for its own sake. To put in terms used by Hülsman (2002, xxxiv - xli) in his introduction to *Epistemological Problems*, one can view preference as either a bilateral relationship between an actor’s mind and the good he is considering, or as a trilateral relationship between the actor’s mind, the one good, and the other good. Hülsman further argues that Mises agreed with him in believing that preferences are bilateral. His argument concerning Mises view is somewhat tenuous, at least in this author’s opinion, although we cannot go into a rigorous critique at this time. For example, he quotes Mises’s *The Theory of Money and Credit* (1953, 39),

“If an individual wishes to make an exchange on an economic basis, he has merely to consider the comparative significance in his own judgement of the quantities of commodities in question. Such an estimate of relative values in no way involves the idea of measurement. An estimate is a direct psychological judgement that is not dependent on any kind of intermediate or auxiliary process.

However, this passage reads more directly as an argument for the now commonly accepted belief that utility is not measurable, as opposed to an argument that it is meaningless to talk about valuing something in some abstract sense distinct from choice. In fact, Mises talks about psychological judgment, which tends to imply that Mises actually thought the exact opposite of what Hülsman argues.

Furthermore, the approach talked by Hülsman is not widely accepted among Neoclassical economists, and it does not find substantial support among Causal-Realists either. To return to Kirzner (1960, 124-128), he discusses the distinction between means,

intermediate ends and ultimate ends, but the very idea of an ultimate end is something you value in and of itself. If preference and value are simply comparisons, then it is meaningless to talk about an ultimate end. As argued above, Mises (1998, 133) also seems to affirm the later view, saying, “The fundamental praxeological insight [is] that men prefer what satisfies them more to what satisfies them less and that they value things on the basis of their utility.” Many modern Causal-Realists also hold this view, for example, Jeffrey Herbener (2012) holds a similar view. In his opinion, it is meaningful to talk about valuing eating an ice cream cone for its own sake. One does not have to talk about valuing eating an ice cream cone more than valuing eating something else.

Rothbard, however, disagrees with these economists and agrees with Hülsman, arguing that one cannot meaningfully talk about people preferring things apart from action, at least in an economic sense of valuing. He (1997, 218) says that “[The concept under discussion] is one example of what we may call the fallacy of ‘psychologizing,’ the treatment of preference scales as if they existed apart from real action.” However, on the very same page he says, “Economics, on the other hand, rests simply on the assumption of the *existence* of ends, and then deduces its valid theory from such a general assumption [emphasis mine].” All that is required to talk meaningfully about value scales apart from action is the existence of ends and the ability to choose between different means to achieve the most desired ends. It is not apparent to this author why not currently having the means always makes the actor unable to determine what he would do if he had them. Nor does there seem to be any reason that people can only comprehend what they value more highly in the process of action. There is some substantial evidence that people find

it difficult to calculate benefits and costs when they do not currently have the means (Diamond and Hausman 1994), but that can be interpreted many different ways including that people value goods relative to a common denominator, and thus find it difficult to compare two goods when they cannot compare them to the common denominator first. It is not evidence that people are always incapable making such comparisons. Rothbard is correct in saying that one cannot distinguish between the accuracy of these approaches based upon what they predict concerning people's actions because they predict the exact same behavior. However, this does not make either of the scenarios wrong or right; one simply cannot choose between the two based on this criterion. This author, for one, is more comfortable with a framework centered upon the individuals' ends, or preferences, than one centered upon demonstrated preference. It allows for much wider-ranged analysis without assuming anything that is not revealed through introspection. Every actor has felt desire for something or satisfaction from consuming something apart from comparing it with something else. For example, as Mises (1998, 12) argued, "A man may prefer sunshine to rain and may wish the sun will dispel the clouds," implying that this was a meaningful statement even though that preference will not affect his action at all. Furthermore, one does not have to talk about people's psychologies, and go beyond the realm of economics, in order to use truths gleaned from talking about them in one's analysis.

This discussion has significant relevance to the topic at hand for if the above point holds, one can meaningfully talk about utility and about the effect of actions upon utility. This and transitive preferences are all that is required to represent people's preferences in

the form of utility functions over some subset of the world's potential states, as shown in the next section. However, it is not enough to allow a utility function to be formed over all possible states, as is normally done. Nor does it imply that doing so would actually be beneficial in analyzing human action. There is a subtle but very important distinction between being able to form a meaningful utility function, and actually wanting to. We will show that the first is possible. However, the second is a substantially more difficult question to answer and is unrelated to the purpose of this paper, and so we will not address it.

Existence of a Utility Function

We will now digress somewhat to show that a utility function can exist over some subset of a person's preferences. This section will be somewhat more technical than the rest of the paper, and we will resume our less technical analysis of Causal-Realist preference theory in the next section.

Consider a decision space (X, \succeq) , where X is some subset of the possible states of the world, and \succeq is an ordinal preference relation that completely orders X . In other words, the person's preferences are complete over X . This is a preordered space because the ordinal preference relation \succeq is reflexive and transitive. Transitivity has been defined above, and reflexivity just requires that for any state of the world α , $\alpha \succeq \alpha$, which clearly holds because otherwise the person would have to strictly prefer a good to itself, which is nonsense. There are two cases to consider here. The first is when the size of X is finite, i.e. there are only finitely-many different goods, and they only come in finitely-many different amounts. This is the normal realm of analysis in the Causal-Realist literature.

They do not usually discuss preference rankings of goods that do not come in discrete amounts. However, this analysis misses an important subset of preferences because both time and probability take on uncountably many different values, and so we must allow the goods that people value to come in uncountably many different amounts.

In the case where the size of X is finite, i.e. all of the possible goods come in finitely-many different values, the existence of a utility function follows relatively easily. Gerard Debreu (1954, 160) showed that “[if] X is a completely ordered subset of a finite Euclidean space [and] for every $x' \in X$ the sets $\{x \in X | x \succeq x'\}$, $\{x \in X | x' \succeq x\}$ are closed (in X) then there exists a continuous, real, order-preserving utility function.” Since X is finite, there are n goods, for some positive integer n , and each of the goods only comes in discrete amounts. Therefore, we are essentially dealing with some finite subset of Z^n , where Z is simply the integers. This is clearly a subset of n -dimensional Euclidean space (R^n). Furthermore, the finiteness of X implies all of its subsets are finite. Therefore, the sets described in Debreu’s definition contain all of their limits, and so they are closed. Therefore, by his theorem a utility function exists.

In the case where X is infinite, the problem becomes substantially more complicated. First, we are still dealing with n goods that are weighted by either their probability or by the length of time until their realization, and both time and probability take on uncountably many different values. The problem in both cases is identical. Therefore, we are essentially dealing with some subset of R^n . If one allows preferences to be continuous and thus allow for indifference, we can use the result of Debreu to get our desired result, thus we assume without loss of generality that $x \not\sim y$ for all $y \in X$. We

will use the theorem proved by Jean-Yves Jaffray (1975, 981). “Let (X, \succeq) be a perfectly separable preordered space; there is on (X, \succeq) a utility function that is continuous in the preorder topology \mathcal{F} .” Because we are not concerned with continuity in this case, we do not have to concern ourselves with what it means to be continuous in the preorder topology \mathcal{F} . However, for the conditions of the theorem to hold, we do need to consider what it means to be a perfectly separable preordered space. (X, \succeq) is a preordered space in this context as it is all contexts. However, what does it mean for a space to be perfectly separable? As defined by Jaffray, a set X is perfectly separable if it contains a perfectly dense countable subset A , that is for any two states $x, y \in X$, where $x \succeq y$, there exists an $a \in A$ such that $x \succeq a \succeq y$ (1975, 981). The reals have this property; we can simply use the rationals (Q) as A because $x \succ y$. Thus, R^n is perfectly separable because we could use the n -dimensional subset of R^n consisting of Cartesian products of rational numbers. Furthermore, Jaffray states that even if we do not allow indifference his theorem still applies. Thus, his theorem applies, and a utility function exists for some subset of a person’s preferences. It is possible, we have neglected some other possibilities concerning a person’s preferences, but those possibilities are rather esoteric in nature and lay far outside the scope of this paper. The interested reader can consult “The Existence of a Utility Function to Represent Preferences” by Trout Rader (1963)

The Admissibility of Indifference Analysis

The other common stipulation area of discussion among Causal-Realists is the applicability of indifference analysis to economic phenomena. Walter Block (2012, 7), representing the majority of Causal-Realists argues,

First, [indifference] is incompatible with human action, choice, decision-making. As a matter of exact language, if one were truly indifferent between two options, there would be no justification for choosing the one of them over the other. Yet, we do choose, all of the time. Hence there is no room for indifference in our intellectual armament.”

Although, Block holds the majority view in Causal-Realist circles there still is some contention in the literature. For example, Ben O’Neill takes the minority view arguing that, “The alternative approach of using non-strict preferences allows indifference to be given a praxeological interpretation and derived directly from non-strict preference relations as a praxeological category.” Israel Kirzner also uses indifference analysis in his work (Mahoney 2011, 1).

Another way of presenting this problem is asking if people’s preferences are continuous. Rejecting indifference analysis is equivalent to rejecting the axiom that people’s preferences are continuous, in most cases.⁴ The reason many Causal-Realists have rejected indifference is because it cannot be demonstrated in action. However, risk and uncertainty present an interesting opportunity for such a demonstration if we use the definition of continuous presented earlier. (If there exist states: A , B , & C , where A is preferred to B , and B to C , then there is a combination with some probability of A and some probability of C where the person is indifferent between that combination and B .) The interesting situation here is infinite divisibility of the probabilities. This infinite divisibility allows one to adapt the definition of continuity used in mathematics. Under normal circumstances, one has to deal with situations of indistinguishability or homogeneity or other amorphous concepts when discussing indifference. However, in

⁴ This is true with both the standard definition of continuity and the one presented here using probabilities. We will present an alternative, substantially weaker, definition of continuity later that does not require accepting the indifference axiom.

this case, if we start with a probabilistic combination and a good where the probabilistic combination is valued significantly less than the other good, we can adjust the probabilities so that the person values the combination more and more until it is valued more than the single good. However, the infinite divisibility implies that one can get the value of the combination as close as one desires to the value of the single good. This is simply the definition of continuity used in mathematics and Neoclassical economics. Rothbard (1997, 230) cunningly shows that such an argument does not lead to measurable utility, as he says, “Measurement...implies the possibility of a unique assignment of numbers which can be meaningfully subjected to all the operations of arithmetic.” Ordinal quantities, whether viewed as ranks as done by the Causal-Realists or equivalently as functions that are unique up unto a monotonic transformation as normally done by Neoclassicals, clearly do not admit such measures. However, this is not an argument that preferences are not continuous, and as far as the author knows, no Causal-Realist has directly dealt with this argument.⁵

The Completeness Axiom

This brings us to the most important part of utility theory for the purposes of this article—the implications of Causal-Realist preference theory concerning the completeness of preferences. As defined earlier, the completeness axiom states that any two states of the world are comparable, that is for states of the world A and B , either $A \succeq B, B \succeq A$ or $A \sim B$. It specifically excludes the possibility that there are two states

⁵ As a side note, the thesis of this paper does not rest upon indifference, and since many Causal-Realists do not admit indifference into their analysis, we shall be careful to avoid making that unproven assumption, and will instead try to spell out where such an assumption would be necessary in building upon our results.

of the world that the person is unable or unwilling to choose between, and not in the sense that they are just indifferent between them. This is precisely the type of assumption that Causal-Realists are wary to make in their analysis, for, as it has been argued, it is inaccurate on both descriptive and normative grounds (Aumann 1962, 440).

The crux of the matter is simply this—is it truly reasonable to assume that the person would be able to decide instantaneously between any two situations even situations that are utterly unlike anything the person has ever faced in real life? Because we are using it to develop our theory of choices under risk, we are only concerned about decisions where the outcome is certain. However, that does not make our task substantially easier, for the complexity of situations that we could face is essentially unbounded. For example, we would have to be able to instantaneously decide between 1) 1001 children in Burkina Faso being shot in 2016 and 2) one world government under the auspices of the Organization of the Petroleum Exporting countries (OPEC) in 2100. As another example, we would have to decide instantaneously which of our two children we wanted to save if they both were drowning, and be able to decide if one was drowning and the other being burned alive, and so on. Now, we may be able to decide in those particular cases. Hoppe (2005, 91) argued that a mother could decide which child to save because she would decide, which does have some validity. However, even if the reader can compare those two examples, it is still a huge assumption to say that every person can instantaneously compare any two possible states of the world, where possible simply means the probability of their occurring is nonzero, although it can be extremely close.

Furthermore, the completeness axiom does not describe reality. Every person has agonized over many decisions over the course of their lifetime. For example, the number of self-evaluations and opportunities for counseling regarding career choices is extremely large and grows every year. Now, these products could be viewed as a means of increasing your information. However, most of the time spend in making a decision like the one concerning a career is not spent simply trying to increase your information but, rather, to evaluate the options available. The decision is far from instantaneous. Similar effects happen in areas such as spouse selection and housing selection.

The inadmissibility of the completeness axiom in the Causal-Realist framework can be further seen from other statements different Causal-Realists have made over the years. For example, Mises (1998) argued that, “The assignment of orders of rank through the valuation is done only in acting and through acting.” He does not go into whether someone can derive valuation between any two goods when one is faced with a decision, nor does he discuss the possibility that someone may value intermediate ends based upon their usefulness in achieving final ends as discussed by Kirzner. However, he is saying that people do not have predetermined valuations of all possible goods, at least in some non-nebulous sense, and it does not seem likely that he would have thought that a person could instantaneously derive such a ranking between any two hypothetical states of the world.

Callahan (2003) in his discussion of indifference says,

Choice does not imply preference over all potentially detectable physical differences between goods. For instance, I do not recall ever picking my clothes based on the spin configuration of the electrons in the garments in question. Nevertheless, I do, every day, pick out clothes with some particular state of electron spin.

However, when if we consider two garments that are identical except for the electron spin, a choice between them implies one of three possibilities. 1) You prefer the one to the other because of the electron spin, which Callahan expressly denies. 2) You are indifferent between the two garments, which is normally denied in Causal-Realist analysis. 3) The two objects are in some sense incomparable at least in the aspect of electron spin. In the real world, we are not likely to encounter two garments that are identical except for their electron spin configuration, but the completeness axiom is strong enough that we have to be able to decide between two such garments, even if they do not actually exist.

Perhaps the clearest rejection of complete preferences was made by Hoppe (1997, 56) when he stated, “For without a complete enumeration of all possible types of actions there can be no knowledge of their relative frequencies. Obviously, no such list of all possible human actions exists, however. We know of a great number of types of action performed then or now, but this list is always open and incomplete.” He was not arguing for incomplete preferences in this passage nor is a complete enumeration technically necessarily required for complete preferences. However, one can reasonably infer from his rejection of a complete enumeration of possible states a rejection of what is essentially a preference order over that complete enumeration.

This rejection of complete preferences is not universal among Causal-Realists. Kirzner (1960, 102) accepted them stating, “in the process of preferring, *all* possible values are placed in an ordered array [emphasis his].” The most important objection to incomplete preferences is that it is impossible to demonstrate an incomplete preference

ranking in action, and this is likely why Kirzner rejected them. Therefore, the question becomes where does the burden of proof lie. However, we are not arguing for the inclusion of an incomplete preference axiom, but rather for the inadmissibility of the complete preference axiom. In other words, we are not arguing that people's preferences have to be incomplete, but rather there is no a priori reason to assume that they are complete, and hence we should do our analysis as agnostics in reference to that question. In our view, which is commonly held in Causal-Realist circles, the burden of proof lies upon those who want to include an assumption, not those who want to reject it. Those who wish to include the axiom have not proven that it should be included, and so the correct response is to reject the assumption, although we do have to be careful to avoid making the opposite, also unproven, assumption that people's preferences have to be incomplete.

Problems with Expected Utility Theory

Before we delve into what a theory of utility in the face of risk would look like based upon the above preference theory, we will examine the literature that has arisen regarding Expected Utility Theory over the past fifty years. Finding a few sparse violations of the theory is not necessarily damning because of human mistakes in reasoning. Perhaps the best way to view the theory is as a description of how people desire to act. There is nothing in the theory that requires people to not make mistakes occasionally; all the theory is saying is that when a person deviates from it, they are making a mistake. However, this method of avoiding the problems is not entirely satisfactory because in many cases even when a person is told they are making a mistake,

that knowledge breeds no conviction, and they still do not change their mind. Secondly, if one is to call this a positive theory of human behavior as opposed to an aid in decision-making, at a bare minimum it would have to be right on average. One of the most important, if not the most important, argument regarding decision-makers' mistakes to arise during the past forty years or so is the Lucas Critique (Kantor 1979, 1424). Lucas argued that decision-makers would be right on average and have the smallest standard deviation possible. Although, this is not macroeconomics nor are we dealing with expectations, per se, actors still have an information constraint and "brainpower constraint" or computational constraint that they need to economize. It is entirely believable that actors conserve their resources and make more decisions that are incorrect as a result. However, although this may allow for some increase in the standard deviation of the population of decisions about the correct decision, it does not explain why the mean of decisions would not be right on average. A logical argument that explains why people deviate from Expected Utility Theory in a relatively consistent fashion in situations where the numeric probabilities are known may exist, but this author does not know of it. Therefore, if someone were to argue that Expected Utility Theory is correct, he would have to explain why the observed deviations are not symmetric around the theory's prediction as well. Essentially, if when people do not follow the theory they are simply making mistakes we would expect the difference between their decision and the standard theory to be randomly distributed. This is not the case.

The critiques of Expected Utility Theory found in the literature come in two varieties: 1) thought experiments and 2) experimental economics. Probably the most

important thought experiment was first developed by Maurice Allais (1953). We will follow the presentation of Chris Starmer (2000). The basic logic runs as follows. Imagine first that you are given a choice between $s_1 = \{\$1M, 1\}$ and $r_1 = \{\$5M, 0.1; \$1M, 0.89; 0, 0.01\}$. That is the first option gives \$1 million for sure, and the second gives a 10% chance of \$5 million, an 89% chance of \$1 million, and a 1% chance of nothing. Next imagine you are given a choice between $s_2 = \{\$1M, .11; 0, 0.89\}$ and $r_2 = \{\$5M, 0.1; 0, 0.09\}$. Allais believed that many people might opt for s_1 drawn by the certainty of the \$1 million, but then choose r_2 because the probability of getting \$1 million is very similar between s_2 and r_2 , and his prediction has played out. This author himself made that choice when he first looked at the problem. However, basically what one chooses s_1 over r_2 is to give up a 10% chance at \$5 million in order to avoid a 1% chance of receiving nothing. This displays an extremely high level of risk aversion, which is contradicted when one chooses r_2 over s_2 . Furthermore, not only do people not make the choice that Expected Utility Theory would predict, even when it is pointed out, many people do not change their minds. The most common, and a highly reasonable, method of avoiding this paradox is to argue that states of the world that people are deciding between are actually more complicated than seen at first glance. The most common explanation of this type says that that if the person does not get the million dollars they will have *regret*, and people value not having regret (Mas-Colell, Whinston, and Green 1995, 180). In other words, regret gives the person negative utility, and people will choose with that in mind.⁶

⁶ For a detailed discussion of an Expected Utility Theory based upon the notion of regret see the article by Graham Loomes and Robert Sugden (1982).

So this paradox in and of itself does not damn the theory, and one would simply have to build the notion of regret into the theory as needed.

This brings us to the second and even more important objection. Rabin and Thaler (2001) bring forth another thought paradox based upon the nature of risk aversion and Expected Utility Theory, and we will carefully follow their exposition. In the context of the standard theory, risk aversion is simply a result of the concavity of the utility of wealth function. In the case of a twice-differentiable function, this simply means that the second derivative is negative. Therefore, if we know how the actor will choose at some level of wealth we can place an upper bound on the amount of risk that she will be willing to undertake at some higher level of risk and/or wealth. There is no reason that a decision-maker has to be risk averse a priori. However, it is an empirical reality that most people are risk averse, and so any theory that purports to apply to all human action has to allow for risk averse actors.

Suppose we have this risk averse actor Johnny who will always turn down a 50-50 bet of losing \$10 or gaining \$11. What else can we say about Johnny's decisions? Specifically, what can we say about Johnny's decision regarding a 50% chance of losing \$100 and a 50% chance of winning some amount \$Y? Rabin and Thaler (2001, 220) state the choices in the following manner.

From the description above, what is the biggest Y such that we know Johnny will turn down a 50-50 lose \$100/win \$Y bet? a) \$110 b) \$221 c) \$2,000 d) \$20,242 e) \$1.1 million f) \$2.5 billion g) Johnny will reject the bet no matter what Y is. h) We can't say without more information about Johnny's utility function.

Did you guess (a), (b), or (c)? Well, then you are wrong. Did you guess (d), (e), or even (f), thinking that we would not have asked unless the answer was ridiculous? Wrong

again. Did you guess (h) thinking that we would need to know more to answer the question? Wrong again. The correct answer is (g). Johnny would never accept a bet of losing \$100 at even odds. This is clearly ridiculous. Many, if not most, people would reject a bet of losing \$10 and gaining \$11 at even odds, but almost no one would refuse a 50-50 bet that involved the risk of losing \$100 but has the possibility of gaining \$2.5 billion.

The easiest way to avoid this is to simply assume that a person can be both risk averse and risk loving at different levels of wealth. Friedman and Savage (1948) actually developed a theory that involved having a utility of wealth function that switched from concave/convex function to explain risk-loving gambling behavior and risk averse insurance preferences. However, Markowitz (1952) convincingly showed that the combined functions lead to a number of patently false predictions.

The other main objection was raised by Richard Watt (2002) who argued that Johnny will turn down the gamble at all possible wealth levels, not just all realistic ones. It is certainly possible then to question Rabin and Thaler's conclusion because that would require an extreme form of risk aversion. However, Rabin and Thaler relax this assumption later in their article, by presenting the following example. Suppose someone turns down a lottery where he loses \$100 or gains \$105 at even odds for any level of wealth less than \$350,000. Then at an initial level of wealth of \$340,000, he would turn down a lottery where he loses \$10,000 or gains \$5.5 million. Although, this is not as ridiculous as the first example, it is still clearly not in line with the way at least some people would act.

According to Ariel Rubinstein (2001, 2-3), Rabin and Thaler's argument rests upon the assumption that the expected utility hypothesis is regarding the final values of wealth, which Von-Neumann and Morgenstern never actually stated. However, Neoclassical economists normally view the theory in this manner, and so even if Von-Neumann and Morgenstern were technically right, the standard interpretation is not.

The third argument lies in the phenomenon of preference reversal. According to Tversky and Thaler (1990, 202-203), people will often choose a bet involving in a large probability of a small gain over a small probability of a large gain, but say that the bet involving the small probability of a large gain is worth more.

Subjects are first asked to choose between two gambles with nearly the same expected values. One gamble, called the H bet (for high chance of winning) has a high chance of winning a relatively small prize (say, 8/9 chance to win \$4), while the other gamble, the L bet, offers a lower chance to win a larger prize (say, a 1/9 chance to win \$40). Most subjects choose the H bet. Subjects are then asked to price each of the gambles. Specifically, they are asked to state the lowest price at which they would be willing to sell each gamble if they owned it. Surprisingly, most subjects put a higher price on the L bet. (In a recent study that used this particular pair of bets, for example, 71 percent of the subjects chose the H bet, while 67 percent priced L above H.)

This phenomenon is distinctly puzzling economists because these two questions are essentially identical. Both questions are asking which one do you prefer. To put in terms commonly used in the Causal-Realist literature, "which of these two subjects are higher on your value scales?" Tversky and Thaler derived their argument from experimental evidence which immediately raises questions from a Causal-Realist perspective because it seems to assume constancy of preferences, especially if there is some time between when the two questions are asked, which there often is (Starmer 2000, 338). However, these experiments do not assume that preferences are constant, but rather they make the significantly weaker assumption that average preferences do not

consistently change in the same direction. It is conceivable that this could happen, but these results have been duplicated enough times to make this author somewhat skeptical of such a claim. There are also questions regarding framing and experimental technique, but the reputability of the researchers who have found this result, such as Kahneman and Tversky (1979) and Lichtenstein and Slovic (1971) makes that explanation unlikely.

This brings us to the fourth critique, which is not commonly found in the literature, but is the most important one. If any of the axioms do not hold, then clearly the expected utility hypothesis does not hold. As explained above, the completeness axiom is at least rather tenuous, and Causal-Realists generally do not accept it. Therefore, the misalignment between the predictions of the theory and the experience in the real world is to be expected.

The other very questionable axiom in Expected Utility Theory is the independence axiom. To quote the definition given previously, for some probability p and some states of the world $A, B, & C$, if $A \succeq B$, then $pA + (1 - p)C \succeq pB + (1 - p)B$. This is axiom that gives preferences under risk most of their structure, and it is unlike anything found in “normal” utility theory. There is no reason to think that a person’s preferences over A and B are not affected by the other goods that he consumes. However, here we are not dealing with the quantities that he consumes, but with quantities that he does not consume. Why would something the person does not consume affect their preferences? If the probability of C is zero, then this result clearly holds. It is not immediately apparent why increasing the probability of C above zero would cause it to

have any effect on the choice between A and B .⁷ On the other hand, to postulate that the independence axiom must necessarily hold in all human action is an extremely strong claim to make. It is extremely hard to prove a universal negative, unless one can argue that the statement under consideration leads to a contradiction. This approach was used to a limited extent by Seidenfeld, Schervish, and Kadane. They show that if one gives up the independence axiom and keeps the completeness axiom then one can end up with Dutch Book situations (1990). A Dutch book situation is simply a situation where there is a gamble where the one person is guaranteed to gain utility at the expense of the other. A well-designed theory of risk would not allow for such situations except as mistakes on the part of one of the actors because the situation would be equivalent to a voluntary exchange where one party gains at the expense of the other, and as the Causal-Realist literature clearly shows such exchanges will not take place. Seidenfeld, Schervish, and Kadane also show that if one drops the well-ordering axiom, which relaxing completeness would do, and retain the independence axiom, such a situation would not occur. Clearly, then if both axioms are relaxed we will not encounter Dutch Book situations either. Thus, we can conclude that the independence axiom has to hold in subsets of preferences where the completeness axiom holds, but it does not have to hold in general. Basically, this author's approach to the independence axiom is that one should not try to build economic theory upon it without making very clear that your theory is based upon a tenuous assumption unless one can show it is true beyond a reasonable

⁷ This argument is derived from the one given by Mas-Colell et al. (1995, 172)

doubt, which has not been done. However, no one has disproven it either, and it does have a certain amount of intuitive appeal.

Utility Theory in the Presence of Risk without the Completeness Axiom

Robert Aumann, who shared the Economics Nobel in 2005, developed the first noteworthy theory of decision-making under risk without some form of a completeness axiom in his article “Utility Theory Without the Completeness Axiom”(1962, 447-448). The focus of his article is explaining how dropping the completeness axiom would affect the relationship between utility theory and preference theory. His main conclusion went as follows,

We still get a utility function u that satisfies the expected utility hypothesis (item (b) above); and u still "represents" the preference order (item (a) above), but now in a weaker sense: as before, if x is preferred to y then $u(x) > u(y)$, but the opposite implication is no longer true. Indeed, since the real numbers are completely ordered and our lottery space is only partially ordered, the opposite implication could not possibly be true. Furthermore, we no longer have uniqueness of the utility [correspondence].

Furthermore, he goes onto argue that one can still solve maximization problems with the utility correspondence, and if one knows the set of all utility functions one can discover the preference order. Furthermore, one is also able to substantially reduce the strength of the continuity, or Archimedean axiom. In Aumann's formulation, for a probability p and states of the world A , B , & C , if $pA + (1 - p)B > C$ for all $p > 0$ then **not** $C > B$. This simply states that if there is some probability of A and some probability of B , where the probability of A is nonzero such that the person strictly prefers that combination to C , then the person weakly prefers B to C . In other words,

when the probability of A switches from miniscule to zero, the person's preference does not switch strongly, although the person may become indifferent. This statement of the continuity principle is especially interesting from a Causal-Realist perspective because the most important problem that Causal-Realists have with continuous preferences is the requirement that one can be indifferent between two states of the world, as explained earlier. However, there is nothing in this presentation that requires the person to be indifferent between two different alternatives, and as a result, one does not have to accept indifference to accept it.

This argument does not, in and of itself, allow us to say much about how people do act under risk. The axioms made by Aumann are simply not restrictive enough; all he showed is that one can form the mapping, if one so desires.

The interesting question in a situation of incomplete preferences is how does an actor respond when two situations are initially incomparable, and yet a choice is required. In evaluating a question such as this, the problem can be examined from two complementary, and not necessarily mutually-exclusive, angles. One can certainly examine it from a psychological perspective, but one can also examine the question from an economics perspective as well. From an economics perspective, the problem is symmetric to how someone chooses a method of satisfying their ends, i.e. they choose the means to achieve their ends according to their ideas. In a case like this, if someone has final ends where the intermediate ends are initially incomparable they will resolve that discrepancy by examining which intermediate ends best allow them to accomplish their final ends. Furthermore, they can also resolve incomparable states of the world by using

the transitivity of preferences. In addition, one can clearly eliminate dominated actions—actions where regardless of what happens a better outcome would occur if another decision had been made. One normally sees this concept in the literature in the form of first-order stochastic dominance or monotonicity, if the reader is familiar with those terms, but we do not need to limit ourselves to areas involving risk. A rational actor would not choose a dominated strategy regardless of whether risk was involved. One most likely sees it discussed most frequently in the risk literature simply because it is so obvious it does not even deserve comment in the non-risk literature.

However, this still leaves a large set of states of the world that are incomparable. If one is comfortable accepting the independence axiom then, one can examine, as Dubra and Ok (2002) did, a more restrictive structure of decision-making under risk. They show that if one can deduce a preference relation using a finite number of steps from the set of comparable states of the world using the independence axiom, then there exists a convex set of continuous utility functions where state A is weakly preferred to the state B if and only if the integral of the utility functions with respect to A is greater than the integral of the function with respect to B (2002, 1061). One would be able to do this if there were only finitely many states of the world, as there is in normal Causal-Realist analysis. One can do it in some other situations as well, but the problem becomes much less tractable. The other statement worth mentioning here is that under these conditions, one derives a set of utility functions not just one, as one does when assuming preferences are complete. In that respect their conclusion is similar to that of Aumann. They also show that if the number of states of the world is finite, then the number of possible utility functions is

finite (2002, 1063). Also, if one accept the independence axiom, this theory is simply a generalization of Expected Utility Theory in that, in the subset of preferences where the preference are complete, it predictions are identical to those made by the standard theory (Aumann 1962, 448).

Consequences of Dropping the Completeness Axiom

To return to an earlier point in the paper, let us examine a few of the consequences of abandoning the completeness axiom on different empirical realities that have called Expected Utility Theory into question. Firstly, the Allais paradox can be explained either by allowing for regret or by arguing the two situations are incomparable. It also helps explain people who initially choose contrary to Expected Utility Theory and then change their minds when further explanation is given, in that having incompleteness introduces a resolving process by which initially incomparable states are reduced to comparable states, but this process presents further opportunity for human error. Therefore, there would likely be more errors. There is some evidence that as people repeat risky interactions their behavior becomes more and more like that predicted by Expected Utility Theory (List 2004), i.e. there is learning involved. This is precisely what is predicted by having incomplete preferences. As people repeat interactions, they become better and better at resolving their preferences in the way that best allows them to achieve their ultimate ends. It also explains evidence for why people might be more likely to overweight or underweight extremely small or extremely large probabilities, as has been often noted empirically (Harless and Camerer 1994, 1273). For since people have finite computational capacity, it could very well be that the economical action in

order to make decision-making less costly is to do exactly that. Furthermore, it helps explain the status quo bias that has been observed quite frequently (Kahneman, Knetsch, and Thaler 1991, 197-199). If one has incomplete preferences, one would expect a person to economize on his time and energy by making comparisons as simple as possible, and one would expect that a comparison to the status quo is often the easiest comparison to make.

To return to the objections raised earlier, the second objection, the one based upon how reasonable risk aversion regarding small bets requires extreme risk aversion regarding large ones, could conceivably still apply in some circumstances. However, it would not hold in general simply because without the completeness axiom we know substantially less about a person's behavior and so we cannot extrapolate near as much from one area to another. Furthermore, there is no reason in this paradigm to think that people make decisions based on final wealth instead of based upon changes in wealth, and as noted earlier this problem rests upon that tacit assumption.

The preference reversal problem could potentially be resolved by arguing that if people came about their decisions in different ways that were both economical methods given the information at hand, they could come to different conclusions, which are both rational. Basically, if the method of reaching the decision is rational, then the conclusion must be rational as well, but because the questions are different it is entirely possible that the methods taken are different and hence the conclusions are different. However, this could allow for intransitive decision-making in some situations, which is rather

problematic from an economics perspective. To examine further where such analysis could apply and what it would actually imply would be a fruitful area of further research

As for the fourth criticism, it simply does not apply. The theory of risk with incomplete preferences does not itself make any assumptions that are questionable from a praxeological perspective, as long as one admits that transitivity is not only universally true but also meaningful in one's analysis. A few Causal-Realists deny this, as discussed in the section concerning transitivity of preferences. One could argue that its reliance on the continuity principle makes it questionable. However, then one would have to consider, which theory one is looking at, as different authors use different continuity axioms (Aumann 1962, 451-452). The independence axiom is highly questionable. However, the theory itself does not include it. One can accept that axiom and thereby limit the theory, but one is by no means required to. The only three assumptions that are not dismissible are 1) That some states of the world are preferred to some others, which is clearly the case. No actor is indifferent or incapable of comparing all possible states of the world. 2) That preferences are transitive. 3) Preferences must follow a very weak form of the continuity axiom that does not require indifference. The first two are obviously true about all human action, and the third stipulation is both very weak and quite intuitive. One could argue that the theory does not imply certain things, such as status quo bias, but that is not a problem with the theory itself, but rather with one's interpretation of its implications.

Conclusion

As we have shown, the standard theory of behavior under conditions of risk, for all its tractability in analyzing issues, does not explain the way people actually behave. Therefore, it is not quality economic theory and should join theories such as the labor-theory of value in the dustbin of now defunct economic models that have historical interest but are not used in mainstream analysis. It may have some value as normatively as a guide to decision-making, or perhaps in some very limited instances, but is not useful as a general theory.

We cannot reject Expected Utility Theory simply on the grounds that it is a mathematical model, for even traditional Causal-Realist utility theory involves assumptions that allow for the creation of a utility function over at least some of the world's possible states. Furthermore, we only need a very weak and as such unobjectionable form of the continuity axiom to allow for the set of utility functions approach developed by Aumann. However, the completeness axiom is highly tenuous and does not align with either introspection or experience, and hence we should reject the axiom, and hence the theory.

If we reject expected utility theory and the completeness axiom, we can develop a theory that is both capable of being logically deduced from a priori truth and is empirically valid. In so doing, we would better understand how people actually act under conditions of risk, which would greatly increase our understanding of economics in areas of risk, including insurance, finance, and even capital theory. We can already say a substantial amount about such behavior, but no complete, coherent theory has been developed. Most of the theories currently in existence are rather nebulous or

psychological in nature, and although there may be some use to them in some instances, a truly economic theory would greatly aid our understanding, and hence further research in this area would be highly fruitful.

On a methodological level, we suggest that economists accept the Causal-Realist refusal to use unfounded axioms without carefully caveating one's results by where the assumption and hence results actually apply. Furthermore, we argue that economists drop the completeness axiom in most, if not all, of their analysis. This also provides a fruitful avenue for further research in both further analyzing the effects of dropping the axiom and analyzing where the assumption actually is valid.

To return to the main point, we conclude that expected utility theory is both apodictically invalid in that it rests upon assumptions that almost certainly false in the sense that they do not apply everywhere, and empirically invalid in that it does not explain how the real world actually works. However, if we drop the completeness axiom we can develop an apodictically valid theory that does explain how the world actually works, and hence greatly strengthen economic analysis.

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