

Results of Inflation Not Proportional to Its Extent

With a Note on Aggregates & Averages

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Abstract. Analysis of the structure of production shows that effects of inflation are not proportional to its extent. They resemble more an infinite sum of disorienting impulses to increase the roundaboutness of production from the perspective of businesses rather a smooth linear shift into greater roundaboutness proportional to the extent of inflation. Even hypothetical slight inflation can force the price system to deviate very greatly from the path it would have taken had there been no intervention, i.e. cause production decisions to change significantly contrary to actual consumer preferences and distribution or availability of capital goods. Harmful effects of intervention are not even approximately linear. And so, minimizing intervention but not eliminating it does not guarantee that the harm arising from intervention is minimal. This agrees with earlier Austrian papers showing that businesses cannot avoid being misled in their production decisions even when they know inflation is taking place. Every burst of inflation, no matter how small, has potential for great decrease in welfare (in the Rothbardian sense).

§1

Introduction

Is extent of change in direction of production, of lengthening of investment chains or increase of roundaboutness of structures of production, which results from inflation, contrary to preferences of consumers, proportional to inflation, which is the increase of the supply of money or money-substitutes, of the supply of loanable funds?

Following Cantillon (1755), Craig (1821), Say (1821), Gossen (1854), Menger (1871; 1888), Hayek (1931; 1932), Hutt (1939; 1956); Mises (1949), Rothbard (1962a; 1962b); Garrison (2001):

1. Producers cannot avoid involvement in business cycles when inflation occurs, even when they know that inflation is taking place by government intervention and know what it causes. They cannot distinguish changes in money-prices of economic goods originating with actual changes in preferences from changes in money-prices of economic goods originating with some individuals receiving additional money or its substitutes earlier than other individuals and spending it according to their own preferences while money and its substitutes held by individuals receiving money later becomes less valuable.

2. Producers cannot do this, because neither can they observe all exchanges taking place in society nor can they observe preferences of consumers directly, and they learn preferences of consumers exclusively through observing changes in money-prices of economic goods, which are ranked among first-order goods, as quantities of money-held, in preferences. They also cannot distinguish changes in market rates of interests, which index personal rates of interest together with risk, caused by additional loanable funds originating with additional saving, or decreases in time-preferences of consumers, from changes in the market rates of interest caused by additional loanable funds originating from additional credit or inflation. Producers lengthen investment chains in order to manufacture greater quantities of economic goods whose market prices had increased and shorten other investment chains, and decreased market rate of interest causes greater lengthening of some production structures than shortening of production structures that are not co-possible. Those individuals or industries who receive money earlier than other individuals or industries gain at their expense; but ultimately, both groups of actors make work decisions that are not sustainable, because they satisfy preferences of consumers less than alternative work decisions they would have made had inflation not taken place.

3. Money-prices of alternative options are used by producers to determine what is more valuable to consumers and therefore how much of what is to be produced, and when, during comparisons of pairs of possible actions that are not co-possible. They respond to changes in money-prices and the market interest rate by changing their investment chains.

4. During inflation, of course, preferences did not change, and lengthening of investment chains or structures of production in response to lower market rates of interest and resulting change in direction of production, of what economic goods are produced instead of those

economic goods that would have been produced otherwise, causes divergence of production from preferences that it was intended to satisfy. Ultimately, there is not sufficient saving for such lengthening of investment chains to take place, and many processes of production cannot be completed to deliver first-order goods from which value of higher-order goods involved in production is derived. Simultaneously production diverges from satisfaction of actual preferences of consumers, and resources are withdrawn from producers that do satisfy actual preferences of consumers, because they are outbid. Some economic goods that are less preferred by consumers are produced in greater quantities than other economic goods that more preferred by consumers. They cannot be sold.

5. Capital is the sum of time-discounted market value of assets, or higher-order goods that are owned by producers that have market prices, indexed by money-prices. Significant quantities of higher-order goods have been wasted, and many higher-order goods that are specific to uses deep within investment chains, after investment chains are shortened, are useless. Decrease in quantity of capital, partly from physical waste of higher-order goods, leaving fewer of them, partly from loss of value of many higher-order goods, consumption of capital, in other words, causes production of first-order goods and higher-order goods that is possible during the near future to be significantly less than it otherwise would have been without inflation.

Is our question, posed earlier, possible to answer solely by analyzing relations between participants of markets? Our question involves determining relations that determine how participants of markets respond to some arbitrary stimulus. In some degree, this sort of question requires analysis of evolution of arbitrary preferences, which determines relative proportions of responses; but that exceeds the scope of the science of economics, and requires taking parts and pieces from physiological psychology.

Actually, there is sufficient information within economics to answer that question, provided we begin with (1) non-quantitative nature of preferences (Cuhel 1907; Bernardelli 1938; 1939; 1952; Mises 1949), and that (2) infinite preferences orders can be decomposed into some series of binary preferences regarding anticipated outcome, taken one at a time, that determine some series isomorphic to it of binary actions originating with binary choices, namely,

to behave in some concrete way distinguishable from every other way one can behave, or to do nothing or to not intervene that way in surroundings, whence to do nothing is also to act, and conscious life is a series of such actions (Condillac 1754a; 1754b; Bernardelli 1936; Mises 1949).

§2

Methods & Procedures

Many reasons exist why preferences and values are fundamentally non-quantitative, some of them logical, some of them physiological. If preferences were quantitative, many contradictions would result: neither choosing nor human action would be possible (Cuhel 1907; Mises 1949). Also, human brains such as they are, in fact, appear to be anticipatory but homeostatic activation-inhibition nets that are hierarchically organized with underlying nets of non-local phenomena at dendrites, and which cannot generally be assumed to be exact sequences of transformations, and where information must be topological, instead of topographical, to have its observed properties (Lashley 1929; Hebb 1949; Hayek 1952; Sokolov 1958; 1960a; 1960b; Pribram 1960a; 1960b; 1991).

Values are exclusively some ordered group which is not a ring. Neither addition, nor multiplication, which is repeated addition, nor subtraction, nor division, which is repeated subtraction, of values is possible.

Let exist, by hypothesis, bundles \mathbf{Q} of distinguishable quantities of different things; but assume that values are non-quantitative. If we follow Pareto (1906a; 1909), then from utility $\mathcal{U}(\mathbf{Q}, I) = 0$ existence of indifference curve $I = \mathcal{Q}\mathbf{Q}$, which supposedly indexes utility, follows by inference, where $M - 1 = N$, so that $\mathcal{U} : 1^1 \cdot \mathbb{R} \times \dots \times 1^M \cdot \mathbb{R} \rightarrow 1^1 \cdot \mathbb{R} \times \dots \times 1^M \cdot \mathbb{R}$ and $\mathcal{Q} : 1^1 \cdot \mathbb{R} \times \dots \times 1^N \cdot \mathbb{R} \rightarrow 1^1 \cdot \mathbb{R} \times \dots \times 1^N \cdot \mathbb{R}$.

This sort of indexing of values is quantitative. It begins with some real numbers, which are placed into some equation that makes them mutually dependent, and so the set of elements determined by this procedure is some group where the ring of arithmetical operations is one true proposition; but that is contrary to hypothesis, which is that values are non-quantitative, and that results in self-contradiction of this sort of analysis, and prevents it from being consistent with physiology, too (Bernardelli 1934; Mises 1949; Rothbard 1956; 1962a; Herbener 1997).

Also, order of labels assigned to distinguishable bundles by ∇I is preserved by continuously increasing or decreasing transformation \mathcal{T} of $\mathcal{Q}\mathbf{Q}$, but order of labels assigned to distinguishable bundles by $\nabla^2 I$ it is not preserved by \mathcal{T} of $\mathcal{Q}\mathbf{Q}$; if so, then what is the point of the whole analysis? (Bernardelli 1938; 1952)

This analysis ultimately fails in other ways, too, because the stimulus that some person encounters *in reality* is some bundle of infinitely many or very many elements; but he or she cannot possibly consume all of them simultaneously. That person must choose what to consume earlier and what to consume later, relatively, within that bundle, which is assigned indifference coefficients; but that is not determined by index I , and must either be arbitrary or pre-determined somehow prior to ordering bundles, according to Pareto (1906b; 1909). Otherwise, then, order in which distinguishable quantities of different things are desired and order in which actions are carried out is not determined, because so long as three or more things are ranked in that way, precise numerical integration of $\nabla \mathcal{Q}\mathbf{Q}$ is not determined.

Is some person really indifferent between drinking his or her soup while standing and then sitting down at his or her table and sitting down at his or her table and then drinking his or her soup? Is preferred consumption order within bundles determined prior to determination of preferred bundles order? How? Aren't functions \mathcal{Q} preferences of people? They are, in this analysis, but conscious people also have strict hierarchical preferences for elements of bundles, which are first-order goods, and such hierarchical preferences are exogenous to functions \mathcal{Q} . They are neither determined nor explained by functions \mathcal{Q} , and cannot be explained by relative positions of index labels that are called indifference curves, because these additional hierarchical preferences exist within bundles that assigned relative numerical labels by functions \mathcal{Q} ; but that means, in this analysis, that people want to want, which is contrary to hypothesis that people have preferences, in fact, because if somebody can want to want, then he or she can want to want to want, and so, preferences can neither be determined nor exist, Leibniz somewhere explained.

And, of course, sets of indifferences cannot lead to action, which is purposeful behavior that is caused exclusively by hierarchical preferences; but preferred consumption order within bundles, and so, binary ranking of subjective important of every pair of wants, is precisely what is not determined by index I (Leibniz 1700; 1717; Mises 1949; Rothbard 1956).

Every stimulus is some marginal quantity, of some good, which is some concrete quantity of some change in one's surroundings at some concrete moment of time that satisfies one's wants, taken as some unit, e.g. total quantities are marginal quantities, too, and in conscious people, sets of these are necessarily isomorphic to sets of preferences, which are non-quantitatively ordered set of elements. (Bernardelli 1938; 1939; 1952; Mises 1949).

Let us consistently label every marginal quantity, which is some physical thing \mathcal{O} with some arbitrary but real and unique numerical label ξ by some map λ , and map $\lambda^{-1} = \psi$ inverts that operation, returning marginal quantity outputs for numerical label inputs. These are « names » of marginal quantities in some mental « language ».

Let $A \neq B \neq C \neq A$, and map F determines hierarchical ordering of sets of marginal qualities and so determines value V . If so, then, at first glance:

$$F(\xi_A + \xi_B) = V_A + V_B = F \xi_C = V_C, \quad \text{or} \quad F(\xi_A - \xi_B) = V_A - V_B = F \xi_C = V_C,$$

$$\text{or} \quad F(\xi_A \cdot \xi_B) = V_A \cdot V_B = F \xi_C = V_C, \quad \text{or} \quad F(\xi_A / \xi_B) = V_A / V_B = F \xi_C = V_C;$$

but that involves self-contradiction: $\psi \varphi(\mathcal{O}_A, \mathcal{O}_B, \mathcal{O}_C) = \psi(\xi_A, \xi_B, \xi_C) = (\mathcal{O}_A, \mathcal{O}_B, \mathcal{O}_C)$ and so

$$\{\psi(\xi_A + \xi_B) = \mathcal{O}_A + \mathcal{O}_B\} \neq \{\mathcal{O}_C = \psi \xi_C\}, \quad \{\psi(\xi_A - \xi_B) = \mathcal{O}_A - \mathcal{O}_B\} \neq \{\mathcal{O}_C = \psi \xi_C\},$$

$$\{\psi(\xi_A \cdot \xi_B) = \mathcal{O}_A \cdot \mathcal{O}_B\} \neq \{\mathcal{O}_C = \psi \xi_C\}, \quad \{\psi(\xi_A / \xi_B) = \mathcal{O}_A / \mathcal{O}_B\} \neq \{\mathcal{O}_C = \psi \xi_C\}.$$

In other words, pairs of numerical labels of things cannot be added, subtracted, multiplied, or divided amongst themselves, of course, because the result of these operations is some other numerical label of something distinguishable from things comprising pairs whose labels were so operated upon; but that is contrary to hypothesis that we indeed started with such-and-such pairs of things instead of that something whose numerical label we obtained by these operations. And so, neither addition nor multiplication nor subtraction nor division of two or more numerical labels within one single map F is possible, nor is addition nor multiplication nor subtraction nor division of two or more hierarchical values, because that means that distinguishable things are identical, contrary to hypothesis that such things are distinguishable.

From this we infer: preferences are not *functions* of numerical labels. There can be no map that happens to be infinitely differentiable at every point of its domain, no map that is analytic, going from one part of its domain to some other part of its domain, for such inputs. Neither are consistent paths between outputs determined by these maps; but then, such maps are neither functions of the real domain nor functions of the imaginary domain (Menger 1954).

What are preferences then? They are maps, because they hierarchically order (e.g. Rothbard 1956; 1962a) different elements comprising some ensemble. Such maps are distributions of numerical labels, because hierarchically ordered sets $\psi F \xi$ and $\psi F \lambda \Theta$ have same elements and they are identical therefore. What cannot be done is exclusively relate different numerical labels by paths that are continuous, smooth, and consistent, by functions.

« We cannot perceive or sense absolute change. We can only perceive or sense relative difference, or change from past configuration » (Boskovich 1755).

Marginal quantities are concrete possible changes to one's surroundings, that one's mind senses, relative to alternative possible changes to one's surroundings that are not co-possible, which satisfy concrete wants, and their values are determined by 1-to-1 mapping onto wants that such changes satisfy, respectively, and they are means to ends; but action or purposeful behavior is often required to cause such changes (Menger 1871). Preferences determine which is chosen, in every binary of set environments in the universe that are sensed or whose sensation is anticipated, one that is such-and-such, and the other that is not such-and-such (Mises 1949).

Every conscious person, at every moment, chooses between two and only two possible behaviors that are not co-possible:

(1) to begin that concrete behavior labeled \mathcal{B} , anticipating by that action to substitute environment \mathcal{E} in place of environment non- \mathcal{B} in the future, because he or she prefers \mathcal{E} to non- \mathcal{E} and so values \mathcal{B} greater than non- \mathcal{B} , anticipating that *to do nothing*, to not intervene in nature, would substitute environment non- \mathcal{E} in place of environment \mathcal{E} in the future, or

(2) to do nothing, to not do that, to begin that behavior labeled non- \mathcal{B} , anticipating by that action to not change one's surroundings in that concrete way, to substitute environment non- \mathcal{E} in place of environment \mathcal{E} in the future, because he or she prefers non- \mathcal{E} to \mathcal{E} and so values non- \mathcal{B} greater than \mathcal{B} . (Mises 1949)

Let \mathcal{V} be set of environments from some one perspective that are possible or anticipated to be possible, are sensed or anticipated to be sensed by somebody's mind, but are not co-possible.

Theorem. Observe: the preferences or the hierarchy of values of that particular observer thus decomposes into some infinity of binary choices, in series, between two logical alternatives, to do \mathcal{B} or to do nothing, or to not do it, which is to do non- \mathcal{B} :

$$V_{\mu} \cdot \mathcal{R} J \cdot \begin{cases} 1 & \text{if } \mathcal{Y}_{\mu} < J < \mathcal{Y}_{\mu+v} \\ 0 & \text{if } J < \mathcal{Y}_{\mu} \cup \mathcal{Y}_{\mu+v} < J \end{cases}$$

and from that we infer that, if identity map $J = \{J, \xi J = \xi = J\xi\}$, and \mathcal{R} is some homomorphism, which preserves order of values assigned to numerical labels of marginal quantities, then

$$F = \begin{cases} \dots & \dots \\ V' \cdot \mathcal{R} J & \text{if } \mathcal{Y}' < J < \mathcal{Y}'' \\ V'' \cdot \mathcal{R} J & \text{if } \mathcal{Y}'' < J < \mathcal{Y}''' \\ V''' \cdot \mathcal{R} J & \text{if } \mathcal{Y}''' < J < \mathcal{Y}'''' \\ \dots & \dots \end{cases}$$

Indeed, map F is a distribution, not a function; but then, what can we infer from it? At first glance there does appear much to infer from some ordered list of elements that preserves its order, regardless of how it happens to be stretched, besides that order itself, and we cannot find out from it relative proportions of responses.

Actually, nevertheless, we can discover some aspects of all of these kinds of responses. How this done is described below.

If the domain of some function is the set of inputs to it for which it has outputs, whose set is the range of it, then its singular points are one sub-set of its domain where that system is not determined, where it has no derivative or where the direction of its outputs disappears. Evolution of the system once such points are reaches is neither determinable nor reversible and information loss, about its past behavior, occurs.

For every function, including those having a real domain, an ensemble of such points can exist, especially if that function is itself an ensemble of points that is not continuous, so that such points are not contiguous, and this ensemble of inputs for which the outputs of that function are singular is the « domain of indeterminacy » of it; but if find some new function having a complex domain with that has derivatives of the first degree in the complex domain at every point in its own domain, that is holomorphic, and which contains outputs for limits of all points in the domain of that previous function, including for those inputs whose corresponding outputs are singular, then such inputs are determinate in the complex domain for that new function, and while rules of differentiation and integration are not true for that previous function, they are true for that new function (Painleve 1900).

Thus, for instance, it is true that « between two truths in the real domain the easiest and shortest path quite often passes through the complex domain ». (Painleve 1900)

In other words, « the shortest and best way between two truths of the real domain often passes through the imaginary domain ». (Hadamard 1945)

Now, we are not dealing with functions, because preferences are not functions, so this shall not actually work; but we can do something very similar. Let $\mathcal{P}DS$ be the ring of all regular analytic functions in open complex-neighborhood $DS \in \mathbb{C}$ of open section $S \in \mathbb{R}$, restricted to some part of the real-line. Let, then, $\mathcal{P}((DS)/S)$ be the ring of all analytic functions in complex neighborhood of S excluding those on S . If $\mathbb{C}_+ = \{ \kappa \in \mathbb{C} \cap \arg \kappa > 0 \}$, $\mathbb{C}_- = \{ \kappa \in \mathbb{C} \cap \arg \kappa < 0 \}$, then equivalence-relation

$$F = [F_+ \text{ if } D \cap \mathbb{C}_+, F_- \text{ if } D \cap \mathbb{C}_-] \in \mathcal{P}((DS)/S)$$

is a generalized function or hyper-function, and $\mathcal{V}(S) = \mathcal{P}((DS)/S) / \mathcal{P}DS$ is the set of every hyper-function defined on S , where A/B is set A excluding set B , not set A divided by set B (Kothe 1952; Sato 1958; 1959; 1960; Imai 1992).

It is possible to de-compose system $F\xi$ into the sum of *hyperfunctions*, which is defined by integration of the corresponding sum of impulses. In fact, the infinite sum of *projections* of values converges in complex-space and *generates* the hierarchy of values in real-space.

Also, if they are *hyperfunctions*, then in general, neither $(F_A \cdot F_B) \xi$ nor $(F_A / F_B) \xi$ are defined (Sato 1958; Imai 1992), which is precisely what is sought for preferences, by the way, to prevent inter-personal comparisons of values. If $i = \sqrt{-1}$, then

$$\begin{aligned}
 F \xi &= \sum_{\mu} V_{\mu} \cdot \mathcal{R} \xi \cdot \begin{cases} 1 & \text{if } \mathcal{Y}_{\mu} < \xi < \mathcal{Y}_{\mu+v} \\ 0 & \text{if } \xi < \mathcal{Y}_{\mu} \cup \mathcal{Y}_{\mu+v} < \xi \end{cases} \\
 &= \lim_{\sigma \rightarrow 0+} \{ F_{+}(\xi + i \cdot \sigma) - F_{-}(\xi - i \cdot \sigma) \} = \lim_{\text{im } \kappa \rightarrow 0+} \{ F_{+} \kappa - F_{-} \bar{\kappa} \} \\
 &= \sum_{\mu} V_{\mu} \cdot \mathcal{R} \xi \cdot \lim_{\text{im } \kappa \rightarrow 0+} \left\{ \frac{F_{+} \kappa}{V_{\mu} \cdot \mathcal{R} \xi} - \frac{F_{-} \bar{\kappa}}{V_{\mu} \cdot \mathcal{R} \xi} \right\} \\
 &= \sum_{\mu} \frac{V_{\mu} \cdot \mathcal{R} \xi}{2 \cdot \pi \cdot i} \cdot \int_{\mathcal{Y}_{\mu}}^{\mathcal{Y}_{\mu+v}} \frac{1}{\varepsilon - \kappa} \cdot d\varepsilon = \sum_{\mu} \frac{V_{\mu} \cdot \mathcal{R} \xi}{2 \cdot \pi \cdot i} \cdot \log \frac{\kappa - \mathcal{Y}_{\mu+v}}{\kappa - \mathcal{Y}_{\mu}} .
 \end{aligned}$$

This is algebraic topology in guise of differential and integral mathematics. At this point, we can return to economics, where the above is going to be meaningful.

§3 Analysis

Observe the instantaneous personal rate of interest, which is determined by the set of time-preferences $F \{C\} / F \{K\}$ of some person. Here, that $F \{C\} / F \{K\}$ is set $F \{C\}$ excluding set $F \{K\}$, not $F \{C\}$ divided by $F \{K\}$. From perspective of some person at some concrete moment, that is obtained by splitting all labeled marginal quantities into two sets with infinitely many elements: consumption goods or first-order goods $\{C\}$, and capital goods, or higher-order goods $\{K\}$.

Ultimately, capital goods can be treated as time-discounted future consumption, which they are used to produce (Rothbard 1962b).

Instantaneous rate of interest at any moment for any arbitrary period is greater than zero, otherwise people would not consume, by putting off consumption forever, which is contrary to hypothesis of them having preferences, because these determine wants that people strive to satisfy by consumption. This rate is $R = \log (1 + F \{C\} / F \{K\}) > 0$, because

$$1 + F \{C\} / F \{K\} = \lim_{N \rightarrow \infty} (1 + R/N)^N = e^R, \quad F \{C\} / F \{K\} > 0,$$

where $F \{C\} / F \{K\}$ must be the discount rate for consumption in the future at the end of the arbitrary period, else more income would be saved and invested into capital and less into consumption than is the action distribution.

Marginal quantities are distinguishable changes in the state of the environment involving concrete quantities of concrete kinds of things at some concrete moment of time, which means that every single numerical label ξ is dated, because identical things at different times are different marginal quantities and have different value.

That means that $\mathcal{R} \xi = \mathcal{F} (R, \dots) \xi$. Although this does not appear to be much its repercussions are substantial, because remember, it's one of the main arguments of $F\xi$, which is some kind of very large series of impulses, a very great series of choices. If that argument changes, then often preferences incredibly different in a very short period of time, and we don't have to know what preferences are precisely to know that this is true. It is invariant to change of preferences.

Let us consider this argument logically now.

Production takes time, and investment started now, production started now, can only yield products in the future, and satisfy future wants. Whence all action, including production, is doubly anticipatory (e.g. Rosen 1985a; 1985b), doubly speculative (e.g. Mises 1949): it is anticipated to satisfy anticipated (future) wants: higher-order goods are used to produce lower-order goods, and first-order goods are means to achieve ends, anticipated means to satisfy anticipated (future) wants, and not ends themselves.

Value of some good is derived from the want it satisfies or its contribution to satisfaction of that want, but higher-order goods are not valued directly in preferences, which only rank first order-goods as direct means to satisfy wants and give pleasure, including quantities of money held for ability to easily purchase other things. Money is an economic good like any other economic good; it is not desired and held without people having reasons for wanting it and holding it (Hutt 1939; 1956). Only through the price system and money-prices can marginal quantities of higher-order goods be compared in value with marginal quantities of lower-order goods in the value scale, and so without market-determined money-prices, rational calculation of how much of each higher-order good to produce and what its value is cannot be determined even assuming that preferences of every person are known to a single entity, which they are not because they are dispersed and cannot be know by any one person (Mises 1949).

Now, suppose inflation takes place. Producers rely on the market rate of interest, which indexes personal rates of interest, to make the incredibly many binary choices regarding whether to lengthen or shorten or keep unchanged these or those particular chains of investment, the whole group of which is the structure of production.

They must anticipate what methods of production shall satisfy consumer preferences, which they do not know because they are in the future, when it is already too late to satisfy them if appropriate production is not begun now, and so, which they must anticipate them also.

They are misled by inflation, by expansion of credit, for instance, or by direct manufacture of additional money by government, which causes the market rate of interest to decrease. It was shown that they cannot avoid being so misled, because they cannot distinguish effects of changes in saving from inflation, especially when legal tender laws are in place.

What happens next? To answer this requires observing the above system. They anticipate entirely different consumer preferences, because how do they anticipate values that consumers hold? Direct observation is not possible. They rely on their own preferences, and must introspectively speculate regarding what future preferences of consumers are. Producers consider the market rate of interest as representative of time-preferences of consumers, which they cannot know directly and which enter into determining all other consumer values. They imagine consumer preferences according to it. That completes the argument. Changing parameters that appear in every single entry of some infinite or very long series of impulses has potential to cause very substantial changes in the result of the series. Impulses are non-linear by definition.

Imagine a list that must be entirely re-organized, because every other element has been assumed to move around in the order. Whether changes of positions in the hierarchy are very slight or very substantial is not relevant, because many slight changes of order are identical to few very greater changes in preferences. Order is entirely relative, and like the well-known monads, every choice is a mirror, in some regard, of every other choice.

And so, producers must anticipate entirely different preferences from what consumers shall actually have in the future, and this shall cause them to make a large number of choices regarding investment chains or what to produce that are contrary to actual consumer preferences in the future. Of course, later, they shall be very surprised to find such a great divergence between what they anticipated consumer preferences where and what they actually turn out to be.

Let us assume that inflation is very great in extent. Anticipations of producers as to what preferences they must try to satisfy will diverge substantially from what these preferences actually are. This shall contribute no small part to the resulting crisis afterwards.

However, what if the inflation was slight in extent? When slightly changing very many choices, the overall result may also be a great change. There is no reason to suspect that changing the parameters of a very large, possibly infinite, series of impulses, every one different from every other, would result in linear change in overall result; but there is, formally, great susceptibility for slight changes in many choices, to lead to something very different from what otherwise would have results. In this case, it would often cause significant changes to the structure of production contrary to actual preferences of consumers, resulting in losses some time later, and contributing part of what define the future situation as a crisis. Damage to welfare, defined as mutual gain arising from voluntary exchanges (Rothbard 1956), would be substantial in both cases, because production shall be misled and pushed by inflation in a direction contrary to the values of consumers, reducing the distribution of mutually beneficial exchanges.

Sometimes, the counter-argument given to such situations, which are non-linear and unstable, is that producers can somehow rely of aggregates or averages to correct their mistakes. In fact, in economics, and in many other systems, use of aggregates or averages is meaningless, for very substantial reasons that are too long to state here and which are detailed in the appendix to this essay, is either totally meaningless or impossible.

When the market rate of interest changes, anticipated preferences of consumers that producers use to determine the direction of investment shall change greatly, whether inflation was slight or great, and production shall be re-directed, not only contrary to actual time-preferences of consumers, but contrary to most other actual consumer preferences also.

Refer to the following figure.

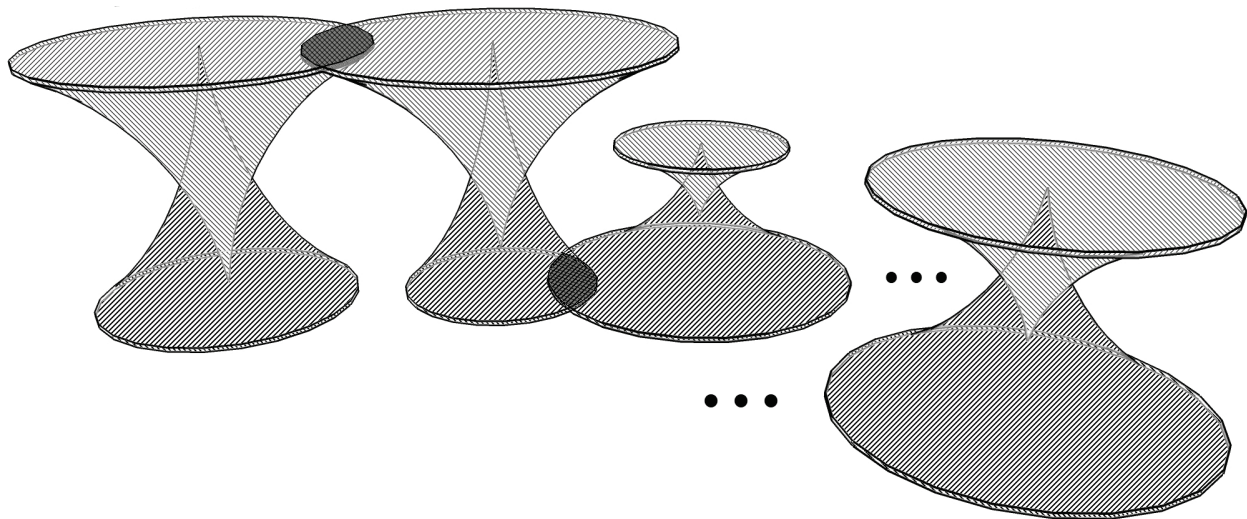


Figure 1. Analysis of the whole market from the perspectives of its parts.

Following Craig (1821), we consider volumes (above) of different marginal quantities of different goods and volumes (below) of buyers, and volumes of buyers of some marginal quantities overlap, if the marginal quantities concerned are complements, and volumes of marginal quantities overlap, if they are substitutes, at that price.

What must realize is that these are neither sums nor volumes in anything except the topological sense. They are topological 1-to-1 mappings of price-slices (and time-slices, which are ignored in the diagram) of marginal quantities, which are distributions again, to price- and time-slices of their respective buyers, which are also distributions, namely, where voluntary exchanges take place. Therefore, welfare is count-wise proportional to number of elements contained in overlaps of such pairs of volumes.

What happens when producers anticipate the wrong preferences and re-direction production along those erroneous lines when they were misled by inflation or otherwise intervention-caused change in the market rate of interest? That situation resembles the first case in next figure, where there is almost no overlap, or many goods that are more expensive are more abundant than buyers and some of them sold, the next case, and this gives the impression to some people on the street that there is a so-called general glut, a superabundance, which is physically impossible; but this is not the whole picture. In fact, people ignore that there is simultaneously a lack of many of marginal quantities of different goods that were produced in quantities far less than could be sold, and yet those are precisely the ones that consumers value more (Say 1821). This is the last case in the following figure. It is this situation, the three cases existing side by side, and disconnected, that characterizes the crisis. There is a great opportunity cost, a regret of foregone want-satisfaction that was otherwise possible (Condillac 1754b), involved in the crisis.

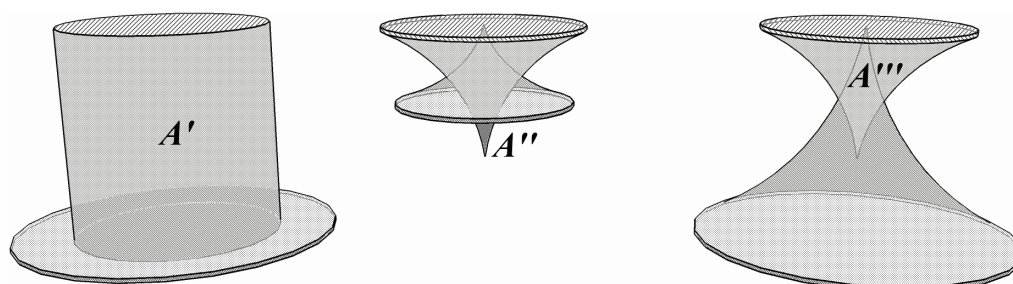


Figure 2. Three possibilities.

In conclusion, this is true, it seems, for every government intervention that changes the market rate of interest, and such analysis might reveal other places where coercive intervention, both slight and great, may cause extended harm, by significantly disturbing people's anticipations of the future and of their relations to each other.

§4 References

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Notes.
Aggregates or Averages?

§1
Introduction.

How do systems evolve, whose parts themselves combine and divide over time, or change in their material composition or mental state, so that over time some parts disappear and new parts appear in different areas of these systems, but where forces that determine such change originate exclusively from individual parts according to local or long-range correlations of their behavior? For instance, (1) systems where « chemical » change over time, e.g. change in composition or number of parts, is distinguishable and exists apart from « physical » change over time, e.g. change in magnitude or direction of forces originating from individual parts or change in position of parts, (2) markets and societies, where actors or their preferences change over time, and where action of every single person influences through past or present actual interaction or future anticipated interaction, from his or her perspective, the environment or preferences of every other person, and the environment of every person along with his or her preferences determines his or her action. Let us label these kinds of systems « index complex ».

Local correlations of behaviors, for instance, include response to a stimulus where the stimulus is actual interaction. Long-range correlations of behaviors, for instance, include response to a stimulus where the stimulus is anticipated interaction or possible interaction.

§2
Method.

Following Rosen (1962; 1963), let time $(T_p, T_Q) \in \mathbb{R} \times \mathbb{R}$ and $T_p \neq T_Q$; if A 's and B 's are sets of numbers and Z 's are sets of things or states of nature, then:

(1) map $R = \{ R\xi T_p = \xi T_Q, (\xi T_p, \xi T_Q) \in Z \times Z \}$ is some « physical system » or « realization » that comprises of causal relations, which takes some one thing or state of nature for its input and gives one thing or state of nature for its output,

(2) map $\varphi : Z \rightarrow A$ is some coherent « encoding » or « language », which consistently maps things or states of nature $\xi T_p \in Z$ to arbitrary numerical labels $\alpha \in A$,

(3) map $M = \{ M\alpha = \beta, \alpha \in A, \beta \in B \}$ is some « logical-mathematical system » or « model » that comprises of inferential relations, which takes one numerical label input and transforms it into one numerical label output,

(4) map $\psi : B \rightarrow Z$ is some coherent « decoding » or « recognition », which consistently maps arbitrary numerical labels $\beta \in B$ to things or states of nature $\xi T_Q \in Z$, so that $R = \psi M \varphi$ and $R \xi T_p = \psi M \varphi \xi T_p = \xi T_Q$, which means that diagram

$$\begin{array}{ccc} Z & \xleftarrow{\psi} & B \\ R \uparrow & & \uparrow M \\ Z & \xrightarrow{\varphi} & A \end{array}$$

commutes.

I argue markets and societies are « index complex » systems. Also, for instance, dendrites, which branch from neurons, and grow amidst glial cells, are « index complex » systems, and so are many chemical systems, too. Such kinds of systems are described below.

Observe system \mathcal{C} , which is ultimately isomorphic to every system that is also « index complex ». Let $X_J K = (\mathbf{X} \in \times_K 1^K \cdot J \mathbb{R})$ and $(V, W) = \mathcal{H} T_Q \times \mathcal{H} T_Q$. If « distance » $\Delta = \kappa_{(V, W)} \cdot \Lambda_J^G C_V \otimes \Lambda_J^G C_W$, then identity map $J = \{ XJ = X = JX \}$, and selection map $\Lambda_J = \{ \Lambda_J \{ J, \mathbf{X}, \dots \} = J \}$. If $U = V \neq W$, then $\ker \Delta (C_V, C_W)(T_p, \mathbf{X}) = \mathcal{H} T_p$ is the set of « nodes » of « net » \mathcal{C} at moment T_p , which are origins of the set of « linkages » $\{ C_V \}$ that exist then and there. Other degrees of freedom are $G = \{ \sigma, \dots, \varpi \}$ and $K = \{ \eta, \dots, \mu \}$.

That is how these kinds of systems behave when observing from some arbitrary but fixed single basis or perspective. If ${}^G C_{V, J} : \mathbb{R} \rightarrow \mathbb{R}$ is some function, so that $\text{dom } {}^G C_{V, J} \subset \mathbb{R}$ and $\text{ran } {}^G C_{V, J} \subset \mathbb{R}$, then let extensive magnitudes

$${}^G C_V = \sum_K {}^G C_{V, K} \cdot {}^{G, K} \mathbf{E} = \begin{bmatrix} {}^G C_{V, \eta} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & {}^G C_{V, \mu} \end{bmatrix},$$

which is are functionals, evolve so that changes of relations between pairs of « linkages » are

$$\nabla^G \mathbf{C}_V(T_P, \mathbf{X}) = \sum_{T_P} \mathcal{F}_{T_P} \left(\bigcup_W \omega_{(V,W)} \cdot {}^G \mathbf{C}_V \otimes {}^G \mathbf{C}_W \right) (T_P, \mathbf{X})$$

where \mathcal{F}_{T_P} is some map that changes over time, and $\omega_{(V,W)} \in \mathbb{C}$.

Above, of course, $\ker \Delta(\mathbf{C}_V, \mathbf{C}_W)(T_P, \mathbf{X}) = \{ (T_P, \mathbf{X}), \Delta(\mathbf{C}_V, \mathbf{C}_W)(T_P, \mathbf{X}) = 0 \}$
 ${}^{G,V} \mathbf{E} \cdot {}^{G,W} \mathbf{E} = 0$, and ${}^{G,V} \mathbf{E} \otimes {}^{G,W} \mathbf{E} + {}^{G,W} \mathbf{E} \otimes {}^{G,V} \mathbf{E} = 0$, from which one infers that
 ${}^{G,U} \mathbf{E} \otimes {}^{G,V} \mathbf{E} = 0$, and composition of mappings is done by their juxtaposition, but inner multiplication is explicitly indicated.

§3 Analysis.

Theorem. One cannot correlate behavior of systems that are « index complex » with aggregates derived from measurement or observation of these kinds of systems, because (1) they cannot be calculated, or (2) they do not exist, or (3) individuals do not respond to changes in aggregates of individual behavior and so they are neither causes nor explanations of it.

Hayek (1931) argued that « neither aggregates nor averages do act upon one another, and it will never be possible to establish necessary connections of cause and effect between them as we can between individual phenomena », that « averages can never form links in reasoning in economic theory », that neither aggregates nor averages are cause of some effect taking place in markets, because individual knowledge and individual preferences determine behavior in markets, and so participants of markets neither know nor respond to aggregates or averages nor to changes thereof, which means that aggregates or averages neither correlate nor explain actions, purposeful behavior, of participants of markets.

Spadaro (1956) demonstrated that aggregates or averages involve fundamentally irreversible loss of information. They do indeed, which is proven here, specifically, that information that is lost by aggregation or averaging is precisely that which determines the evolution of the entire system.

Also, « economic action is ultimately dependent for explanation on individual differences, but employment of averages puts us out of reach of such explanation simply by under-stating these differences. For averages, by their nature, can only minimize if not entirely eliminate, differences; they cannot magnify them. There is thus no possibility of drawing comfort from any compensating effect of large numbers. Distortion brought into play by the use of averages cannot, ironically, itself be « averaged out ». Doubt concerning the validity of aggregates and averages is a dagger aimed straight at the heart of much current empirical research and statistical analysis in economics ». (Spadaro 1956)

Actually, very often, aggregates of « index complex » systems, for instance, markets, cannot be calculated by observation and measurement of behavior or do not exist. They are not causes and so not explanations of actions of participants of markets, because then participants of markets respond to neither aggregates nor averages, which they cannot know, nor can things that cannot be calculated or do not exist be correlated and serve as explanations of behavior of participant of markets.

In these kinds of system, we observe exclusively some arbitrary sub-set of the set of changes of relations between pairs of individual parts of the whole system, namely, $\Pi \in \{ \nabla^G C_{\nu}(T_p, \mathbf{X}) \}$, because we can observe relative differences but nothing absolute.

Following Truesdell (1966), from some arbitrary but fixed single basis or perspective, if $T_p < T_Q$, then averages over time of law of motion ${}^{\tau} \mathcal{L} : A \rightarrow B$, where \mathcal{G} is the set of distinguishable kinds of manipulation $\mathcal{M}_{\mathcal{G}}$ of ordered pair $(\alpha, \beta) \in A \times B$ of numerical labels that are results of measurement, are kinds of aggregates,

$$\widehat{\mathcal{M}_{\mathcal{G}}}(T_p, \mathbf{X}) = \lim_{T_Q \rightarrow \infty} \frac{1}{T_Q - T_p} \cdot \int_{T_p}^{T_Q} \mathcal{M}_{\mathcal{G}} {}^{\tau} \mathcal{L} \mathbf{X} \cdot d\tau$$

where existence of $\widehat{\mathcal{M}_{\mathcal{G}}}(T_p, \mathbf{X})$ for every T_Q implies that $\widehat{\mathcal{M}_{\mathcal{G}}} \mathbf{X} = \widehat{\mathcal{M}_{\mathcal{G}}}(T_p, \mathbf{X})$, which means that averages over time, where they exist for every moment of time, do not change over time, because changing moment of time when measurement is done and they are calculated does not result in different time-averages, since every moment of time already entered into them.

Let ${}^{\tau}\mathcal{L}\mathbf{X} = \nabla^G \mathbf{C}_V \mathbf{X}$; but then, averages of time of \mathcal{M}_g cannot be calculated, because $\nabla^G \mathbf{C}_V \mathbf{X}$ changes over time according to index V , and index V in time-averages

$$\widehat{\mathcal{M}}_g(T_P, \mathbf{X}) = \lim_{T_Q \rightarrow \infty} \frac{1}{T_Q - T_P} \cdot \int_{T_P}^{T_Q} \mathcal{M}_g \nabla^G \mathbf{C}_V \mathbf{X} \cdot d\tau$$

is neither already determined nor can it be observed without laws of individual behavior, which are not known when averaging observed individual behavior over time.

Let averages over time exist by hypothesis for systems that are « index complex » for every moment of time. Then, if somehow $\nabla^G \mathbf{C}_V \mathbf{X}$ was given, then $\widehat{\mathcal{M}}_g(T_P, \mathbf{X})$ does change over time, because V does change over time; but that is contrary to hypothesis that averages over time exist for systems that are « index complex » for every moment of time, because if they do, then averages over time do not themselves change over time. And so, averages over time necessarily do not exist for « index complex » systems for every moment of time, and because this is true of other intervals of time shorter than infinite time, averages over time do not exist for « index complex » systems at all.

Actually, if system \mathcal{C} of laws of individual behavior is replaced by whatever system \mathcal{H} that comprises of hypothetically correlated aggregates of individual behavior, namely,

$$\widehat{\mathcal{M}}_g(T_P, \mathbf{X}) = \sum_G \sum_V \mathcal{M}_g \nabla^G \mathbf{C}_V(T_P, \mathbf{X})$$

without knowledge of laws of individual behavior, for instance, averages, then such aggregates cannot be calculated, because they are not determined in system \mathcal{H} , where index V also evolves over time but is determined exclusively by laws of individual behavior that are ignored by system \mathcal{H} . These laws exist solely in system \mathcal{C} .

Sums over indices that cannot be determined cannot themselves be determined. These aggregates, where they exist, are determined in system \mathcal{C} ; but there, in system \mathcal{C} , laws of individual behavior already exist. Precisely these laws determine index V that is required to calculate aggregates and so reasons to calculate them are absent in system \mathcal{C} .

When observers construct aggregates of individual behavior of « index complex » systems, what they in fact do is arbitrarily choose index V according to some observer's arbitrary *opinion* of what aggregates of individual behavior are, not according to fact, because observers know their own opinions concerning what they think index V is, or can find out other people's opinions of index V , but they cannot possibly sum over what index V actually is. Why? In fact, index V , in systems that are « index complex », is known for some moments of time exclusively when laws of individual behavior are known and not otherwise. It determines, in turn, laws of individual behavior at other moments of time; but that is contrary to hypothesis that aggregates of individual behavior, once calculated, allow us to discover the state of the system without knowledge of laws of individual behavior.

Where they exist, in physics, aggregates are objectively determined, and there they describe nature, but in economics, they are subjectively determined, and so they describe arbitrary preferences of observers of nature but say nothing about nature itself.

For instance, we cannot aggregate market demand by measurement for some part of the whole market, because we cannot know who is participating in it at different moments unless we already know laws of individual behavior *a priori* without measurement. So, if somebody had arrived at some aggregate of it, then who is participating in that part of the whole market at different moments of time must have been arrived at by arbitrary opinion. This aggregate, then, is subjective, determined by observer's preferences, not some objective datum that actually describes market demand.

In other words, history of acting persons comprises entirely and exclusively of unique events; but aggregates or averages, which can be calculated or exist solely for ensembles, cannot be calculated by observation and measurement of unique events or do not exist where all events are unique events, where there are no constant quantities but exclusively variable quantities, where individual differences are what determine behavior, and so analysis of actions of persons cannot be done by calculation or correlation of aggregates or averages. (Mises 1957)

All other kinds of aggregates or averages are ultimately involve summing over time or themselves are functions of time-averages, because they comprise of measurements of parts of some ensemble or set that are gotten by observation over time; but sums over time fundamentally do not exist for « index complex » systems, and so, those other kinds of aggregates or averages do not exist for systems that are « index complex ».

§4
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