## Good Morning and Welcome to the 2009 Calcu-Solve Competition!

## We hope you have a challenging and successful day! <br> While we are waiting for all the teams to arrive, please:

1. Put your coats and lunches in the designated areas where your team sponsors can keep an eye on them. The only things you need to take to your team's table are your sharpened pencils, calculators, and scrap paper (if you brought any.)
2. Make sure your team sponsor has completed the yellow registration/scoring card that is in the folder he/she received when you arrived. This card should be filled out completely and given to Mrs. Hoedeman at the Registration Desk.
3. Make a nametag for everyone in your group. Include your first and last names, school district, school name, and team number on the tag. Please wear the nametag during the entire competition.
4. Read over the information in the folder with your team sponsor. The rules and scoring procedures are explained. We will review these briefly just before the competition begins.
5. Each person on your team should take one stapled packet of individual answer sheets from the team folder and complete all the information on EVERY sheet. Print neatly and accurately! Your team number is displayed on the sign at your table and on your team folder.
6. Begin to practice for the competition by working on the Warm-Up Questions that are in your folder. We will go over the answers to these questions just before we begin the actual competition questions.
7. If you need help or further direction, please find a Boyce Ambassador or see Mrs. lpedeman, Mr. O'Roark, Mrs. Hamel, or Mrs. Husak.

## Relax, Have Fun, and Good Luck!

## Dr. Robert L. Furman

QuickTime ${ }^{\text {TM }}$ and a
TIFF (Uncompressed) decompressor are needed to see this picture.

Executive Faculty
Program Director
Educational
Administration and Supervision

## Mrs. Karen Brown

Principal at Boyce<br>Middle School

Mission - to create a stronger population of new engineering-, science- and mathematics-educated individuals that will ultimately increase and improve the pool of engineers and scientists seeking to tackle the tough and ever more complicated technical problems facing our nation and the world.

## Warm-Up Answers

1. $1 / 4+1 / 4+1 / 4=18$ minutes; so $6+6+6=18$ minutes; so it will take her $\mathbf{6}$ more minutes to walk the rest of the way home.
2. Ursula-3m-Alma-3m-Cathy-2m-Lani-2m-Isabel-2m-Betty
3. $\div 2, \div 3, \div 4, \div 5$, so $\underline{30}$
4. 11 bicycles $(2 \times 11=22)$; and 4 tricycles $(4 \times 3=12)$; and $22+12=34$ wheels
5. to the game: cars
from the game: cars 4
so, there were 22 friends and 5 cars.
6. $3: 3,6,9,12,15 \ldots 30 \ldots 45 \ldots 60$

5: $5,10,15, \ldots 30 \ldots 45 \ldots 60$
$15 \div 7 \neq R 4 ; 30 \div 7 \neq R 4 ; 45 \div 7 \neq R 4 ; 60 \div 7=R 4$
7. Friend: $\begin{array}{llllllll}7 & 2 & 3 & 4 & 5 & 6 & 7 & 8\end{array}$

Trades: $\quad 7+6+5+4+3+2+1+0=\underline{\mathbf{2 8}}$ trades altogether.
8. $\$ 700=70,000$ cents; $25 \phi+50 \phi+100 \phi=175 \phi$;
$70,000 \div 175=\underline{400}$ of each coin.
9. 30 minutes $=12$ minutes +12 minutes +6 minutes; $1+1+0.5=2.5$ times as far; so, Sue can walk $2.5 \times 1.56$ miles $=\underline{3.9}$ miles
10. $7 / 12=14 / 24$ and $5 / 8=15 / 24$, so $15 / 24-14 / 24=\underline{1 / 24}$

## Thank You from

 Mrs. Hoedeman and Mr. O'Roark- Mrs. Obrosky
- Ms. Fleckenstein
- Mrs. Heins
- Mrs. Cayuela
- Mrs. Husak
- Mrs. Hamel


## SCHEDULE and EXPLANATION OF SCORING

There will be eight Individual Questions - \#s 2, 3, 4, 5 and 7, 8, 9, 10. You will be given 5 minutes to earn 5 points for a correct answer on each Individual Question, or you may wait for a clue, work an extra 2 minutes and earn 3 points for a correct answer on each Individual Question.

There will be two Group Questions - \#s 1 and 6. Your team of students will be given $\mathbf{7}$ minutes to earn 10 points for a correct answer on each Group Question.

We will complete Group Question \# 1, Individual Questions \# 2, 3, 4, and 5 and Group Question \#6. Then we will take a short break and announce half-time scores.

After the break, we will complete Individual Questions \# 7, 8, 9, and 10.
Following Individual Question \# 10, we will break for lunch.
If necessary, "tie-breakers" will follow lunch.
Final scores will be announced and awards will be presented after ties are broken.

Estimated concluding time is 1:30 p.m.

## Guidelines for Tie-Breaking Situations Individual Tie-Breakers*

1. In the event of a tied individual score, a sudden death question will be given to those participants who are tied. If an answer is turned in and it is incorrect, the person may return to his/her seat and continue to work on the problem. The first person with a correct answer within a 5-minute time limit will be declared the winner. If at the end of 5 minutes, no one has submitted a correct answer ...
2. ...another sudden death question will be given and step \# 1 will be repeated. This procedure will be followed until a winner is determined.
*These rules will be used to determine first, second, third, and tenth place individual winners.

## Group Tie-Breakers**

Group tie-breakers will be handled in the same fashion as individual except that the entire group will participate.
*These rules will be used to determine first, second, and third place teams only.

## Sample Problem

Given the following pattern: $1,1,2,3,5,8 \ldots$ what is the first perfect square greater than one to occur in the sequence?

Sample Problem - Clue

Given the following pattern: 1, 1, 2, 3, 5, 8 what is the first perfect square greater than one to occur in the sequence?

Clue:There is a famous pattern known as the Fibonacci Sequence where each number after the first two numbers is the sum of the preceding two numbers.

## Official Competition

## Group Question

Set timer for 7:00
D

## Boyce Ambassadors

Please pass out the 1st group question face down and the 1st group answer sheet.

Question 1 - Group

What is the smallest number that is divisible by all of the integers from 1 through 10 ?

## Question 1 Answer

What is the smallest number that is divisible by all of the integers from 1 through 10 ?
This question is all about factors.
To be divisible by 9 it must have two 3s
To be divisible by 8 it must have three 2s
We must have a 7 and a 5 because they are prime.
That is all we need $2,3,4,6$, and 10 are already covered because they are contained in or formed by the factors of $5,7,8$, and 9 .
Therefore the smallest number that is divisible by the numbers $\mathbf{1 - 1 0}$ is....

## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 1st individual question face down.

## Question 2

Jack drives a gas-guzzling SUV 15,000 miles a year. It averages 10 mpg . His wife, Jill, drives an economy car the same distance. It gets 30 mpg . The couple wants to use less gasoline. They can afford to trade Jack's clunker for an SUV that gets 13 mpg or trade Jill's sensible car for a hybrid that gets 60 mpg . To the nearest gallon, what is the maximum number of gallons the couple could save if they can only trade in one car?

## Question 2 Clue

Jack drives a gas-guzzling SUV 15,000 miles a year. It averages 10 mpg . His wife, Jill, drives an economy car the same distance. It gets 30 mpg . The couple wants to use less gasoline. They can afford to trade Jack's clunker for an SUV that gets 13 mpg or trade Jill's sensible car for a hybrid that gets 60 mpg . To the nearest gallon, what is the maximum number of gallons the couple could save if they can only trade in one car?

## Clue:

CLUE: Jack and Jill just saved a boatload of money by switching to Geico!

## Question 2 Answer

Jack drives a gas-guzzling SUV 15,000 miles a year. It averages 10 mpg . His wife, Jill, drives an economy car the same distance. It gets 30 mpg . The couple wants to use less gasoline. They can afford to trade Jack's clunker for an SUV that gets 13 mpg or trade Jill's sensible car for a hybrid that gets 60 mpg . To the nearest gallon, what is the maximum number of gallons the couple could save if they can only trade in one car?

## Solution:

The 10 mpg SUV uses $\mathbf{1 5 0 0}$ gallons of gas yearly ( $15,000 / 10=1500$ ). The 13 mpg SUV would use about 1154 gallons ( $15,000 / 13$ = about 1154). The economy car uses 500 gallons of gas ( $15,000 / 30=$ 500). The hybrid would use 250 gallons (15,000/60 $=250$ ).
So, if the couple trades Jack's SUV, they would save 346 gallons yearly ( $1500-1154=346$ ). But if they trade Jill's car, they would save only 250 gallons

$$
(500-250=250) .
$$

## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 2nd individual question face down.

## Question 3

Jeans are on sale for $25 \%$ off. Customers may receive an additional mystery discount off the already discounted price by popping a balloon. Jessica wants to buy the jeans on sale and use the balloon discount for an additional discount. What mystery percent would she have to get so that the total discount is $50 \%$ off the original price?
Round your answer to the nearest whole percent.

## Question 3 Clue

Jeans are on sale for $25 \%$ off. Customers may receive an additional mystery discount by popping a balloon. Jessica wants to buy the jeans on sale and use the balloon discount for an additional discount. What mystery percent would she have to get so that the total discount is $50 \%$ off the original price? Round your answer to the nearest whole percent.

- CLUE: Pick a price.


## Question 3 Answer

Jeans are on sale for $25 \%$ off. Customers may receive an additional mystery discount by popping a balloon. Jessica wants to buy the jeans on sale and use the balloon discount for an additional discount. What mystery percent would she have to get so that the total discount is $50 \%$ off the original price? Round your answer to the nearest whole percent.

## Solution:

- Let the original jeans cost $\$ 100$, so the $\mathbf{2 5 \%}$ off sales price is $\$ 75$. If I want the final price to be $50 \%$ off $\$ 100$, I need the final price to be $\$ 50$. To get the sale price of $\$ 75$ down to $\$ 50$, I need to subtract $\$ 25$ which is $1 / 3$ of $\$ 75$. I need a coupon for


## 1/3 off or 33.3\%.

## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 3rd individual question face down.

## Question 4

Jenna usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on for 7 hours, and during that time she has used it for 60 minutes. If she doesn't talk anymore but leaves the phone on, how many more hours will the battery last?

## Question 4 Clue

Jenna usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 7 hours, and during that time she has used it for 60 minutes. If she doesn't talk anymore but leaves the phone on, how many more hours will the battery last?

## CLUE: Never drive while texting!

## Question 4 Answer

Jenna usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 7 hours, and during that time she has used it for 60 minutes. If she doesn't talk anymore but leaves the phone on, how many more hours will the battery last?

## Solution: Jenna used her cell phone

 constantly for 1 out of 3 hours using $1 / 3$ of the battery. $1 / 3$ of 24 is 8 . That leaves $24-8=16$ hours remaining. The phone has been on for an additional 8 hours leaving $16-6=10$ hours of battery life left.
## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 4th individual question face down.

## Question 5

For every 75 cars made at a factory, $1 / 3$ are red, 20 are blue, and the rest are green. Seven out of every 10 blue cars have GPS's installed and $40 \%$ of the red cars have GPS's. No green cars are produced with GPS's. If the factory produced 960 cars with GPS's, how many green cars did it produce?

## Question 5 Clue

For every 75 cars made at a factory, 1/3 are red, 20 are blue, and the rest are green. Seven out of every 10 blue cars have GPS's installed and $40 \%$ of the red cars have GPS's. No green cars are produced with GPS's. If the factory produced 960 cars with GPS's, how many green cars did it produce?
Clue:

## There are more than 960 cars.

## Question 5 Answer

For every 75 cars made at a factory, $1 / 3$ are red, 20 are blue, and the rest are green. Seven out of every 10 blue cars have GPS's installed and $40 \%$ of the red cars have GPS's. No green cars are produced with GPS's. If the factory produced 960 cars with GPS's, how many green cars did it produce?

## Solution:

If $1 / 3$ are red, red $=25$; blue $=20$ and green $=75$ $-(20+25)=30.40 \%$ of red have a GPS, or 10 out of the $75(.4 \times 25=10)$ and $7 / 10$ or .7 of blue have a GPS, or $.7 \times 20=14$. That means 24 of the 75 cars have a GPS. From here use proportions:
$\frac{24}{75}=\frac{960}{x}$
$\frac{30}{75}=\frac{x}{3000}$
$x=3,000$ cars $\quad x=1200$ Green cars

## Group Question

Set timer for 7:00
D

## Boyce Ambassadors

Please pass out the 2nd group question face down and the 2nd group answer sheet.

## Question 6 - Group

Billy-Bob is playing darts and needs to hit the gray portion of the target. Unfortunately his aim isn't very good and his dart randomly strikes anywhere on the target. The center of the smallest circle is 4 inches across and each ring is 2 inches wide. What is the probability that the dart strikes the gray portion of the target as a fraction in simplest form?

## Question 6 Answer

Billy-Bob is playing darts and needs to hit the grey portion of the target. Unfortunately his aim isn't very good and his dart randomly strikes anywhere on the target. The center of the circle is 4 inches across and each ring is 2 inches wide. What is the probability that the dart strikes the grey portion of the target as a fraction in lowest terms?

## Solution:

$A=\pi r^{2}$ To find the area of the rings simply find the area of the larger circle and subtract the area of the next smaller circle.

$$
\begin{array}{ll}
64 \pi-36 \pi=28 \pi & 16 \pi-4 \pi=12 \pi \\
28 \pi+12 \pi=40 \pi & \frac{40 \pi}{64 \pi}=\frac{5}{8}
\end{array}
$$

## Snack Time

 We will dismiss by tables. Please wait until you are dismissed and then take 1 cookie and 1 drink.
## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 5th individual question face down.

Question 7

On a coordinate plane the distance from the origin to $(0,1)$ is 1 cm . A polygon graphed on this plane has vertices of $(8,10),(0,4)$, $(0,-2),(12,-2)$, and $(12,4)$. What is the area of this polygon?

## Question 7 Clue

What is the area of a polygon with vertices at $(8,10),(0,4),(0,-2),(12,-2)$, and $(12,4)$ ?

## Clue:

## Draw a picture.

## Question 7 Answer

What is the area of a polygon with vertices at $(8,10)$, $(0,4),(0,-2),(12,-2)$, and $(12,4)$ ?

## Solution:

$$
A=\ln A=6 \times 12=72
$$

$$
A=\frac{1}{2} b h \quad A=\frac{1}{2} 12(6)=36
$$



Total area is $72+36=108 \mathrm{~cm}$. squared.

## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 6th individual question face down.

## Question 8

A crate filled with oranges weighs 10 pounds. The cross country team ate two-fifths of the oranges. The remaining oranges and crate together weighed 7 pounds. Find the weight of the empty crate.

## Question 8 Clue

A crate filled with oranges weighs 10 pounds. The cross country team ate two-fifths of the oranges. The remaining oranges and crate together weighed 7 pounds. Find the weight of the empty crate.
Clue:
The team ate 3 pounds of oranges.

## Question 8 Answer

A crate filled with oranges weighs 10 pounds. The cross country team ate two-fifths of the oranges.
The remaining oranges and crate together weighed 7 pounds. Find the weight of the empty crate.

$$
c+r=10
$$

Solution:

$$
c+\frac{3}{5} r=7 \begin{aligned}
& \text { Therefore the crate } \\
& \text { weighs } 2.5 \text { pounds }
\end{aligned}
$$

$$
\begin{aligned}
& \frac{2}{5} r=3 \\
& r=7.5
\end{aligned}
$$

## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 7th individual question face down.

Question 9

A phone number is randomly selected from a phone book. What is the probability that the last 3 digits of the phone number will all be prime? Express your answer as a fraction in lowest terms.

## Question 9 Clue

A phone number is randomly selected from a phone book. What is the probability that the last 3 digits of the number will all be prime? Express your answer as a fraction in lowest terms.

## CLUE: 1 is not a prime number.

## Question 9 Answer

A phone number is randomly selected from a phone book. What is the probability that the last 3 digits of the number will all be prime? Express your answer as a fraction in lowest terms.

Solution: There are 4 single digit prime numbers and thus $4 \times 4 \times 4=64$ that have all of their digits as prime numbers out of $\mathbf{1 0 0 0}$ possibilities. This simplifies to

125

## Individual Question

## Set timer for 5:00

Set timer for 2:00 for the clue

## Boyce Ambassadors

Please pass out the 8th individual question face down.

## Question 10

Mrs. Welch bought a bunch of seedless red grapes at the local market and placed them in a fruit bowl on the kitchen table. They looked so good that she ate $1 / 8$ of them before going outside to work in her flower garden. When Mr. Welch passed the grapes on the way to the refrigerator, he stopped and ate $2 / 7$ of the remaining grapes. Their daughter, Sally, who was working diligently on her math homework, decided to stop for a break to get a snack and something to drink. Noticing how appetizing the grapes looked, she ate $1 / 5$ of the grapes that remained in the fruit bowl. Returning from football practice, their son, Bob, immediately gulped down $3 / 4$ of the remaining grapes. When Mrs. Welch returned from weeding her flower garden, she was surprised to see only 6 grapes left in the fruit bowl. How many grapes were in the original bunch?

## Question 10 Clue

Mrs. Welch bought a bunch of seedless red grapes at the local market and placed them in a fruit bowl on the kitchen table. They looked so good that she ate $1 / 8$ of them before going outside to work in her flower garden. When Mr. Welch passed the grapes on the way to the refrigerator, he stopped and ate $2 / 7$ of the grapes that Mrs. Welch left. Their daughter, Sally, who was working diligently on her math homework, decided to stop for a break to get a snack and something to drink. Noticing how appetizing the grapes looked, she ate $1 / 5$ of the grapes that remained in the fruit bowl. Returning from football practice, their son, Bob, immediately gulped down $3 / 4$ of the remaining grapes. When Mrs. Welch returned from weeding her flower garden, she was surprised to see only 6 grapes left in the fruit bowl. How many grapes were in the original bunch?

## Clue:

## You could work backwards.

## Question 10 Answer

Mrs. Welch bought a bunch of seedless red grapes at the local market and placed them in a fruit bowl on the kitchen table. They looked so good that she ate $1 / 8$ of them before going outside to work in her flower garden. When Mr. Welch passed the grapes on the way to the refrigerator, he stopped and ate $2 / 7$ of the grapes that Mrs. Welch left. Their daughter, Sally, who was working diligently on her math homework, decided to stop for a break to get a snack and something to drink. Noticing how appetizing the grapes looked, she ate $1 / 5$ of the grapes that remained in the fruit bowl. Returning from football practice, their son, Bob, immediately gulped down $3 / 4$ of the remaining grapes. When Mrs. Welch returned from weeding her flower garden, she was surprised to see only 6 grapes left in the fruit bowl. How many grapes were in the original bunch?

## Solution: You could have worked backwards and found the solution. However, I like this better.

## $\frac{7}{8} \bullet \frac{5}{7} \bullet \frac{4}{5} \cdot \frac{1}{4} \bullet x=6$

Everything cancels out leaving $\frac{1}{8} x=6$
There were 48 grapes in the original bunch.

