

2000-2001 CalcuSolve Bowl

Grades 7 & 8

Allegheny Intermediate Unit

Pittsburgh, Pennsylvania

In a group of 48 students that recently took a math test, the average score was 90 points. Some of the students received a score of 100 points, and 20 students each received a score of 80 points, and all the others received 90 points each. How many students received a grade of 100 points?

Round 1 Solution:

Let

X = the number of students that received 100 points.

Since 20 students received 80 points each, then

$48 - X - 20$ = the remaining number of students that received 90 points.

The average score of 90 can be calculated from the following equation.

$$\frac{X(100) + 20(80) + (48 - X - 20)(90)}{48} = 90$$

Which gives $X = 20$.

Answer: 20 (students)

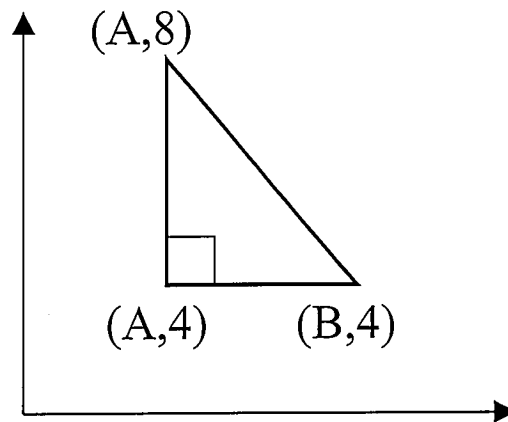
What is the area of a triangle with vertices at the points $(A,4)$, $(A,8)$, and $(B,4)$? Express your answer in terms of A and B , and assume that B is greater than A .

Clue: The base of the triangle has a length of $B - A$.

Round 2 Solution:

Based on the given vertex coordinates, the triangle will roughly appear as shown below. Its base length is $B - A$, and its height is 4. And its area is

$$\begin{aligned}\text{Area} &= \frac{1}{2} (\text{Base})(\text{Height}) = \frac{1}{2}(B-A)(4) \\ &= 2(B-A)\end{aligned}$$



Answer: $2(B-A)$ or $2B-2A$

Find the value of y , given that

$$3^{62} + 3^{60} = 10(3^{y+1})$$

Clue: $3^{62} = 3^{60}3^2$

Round 3 Solution:

Starting with

$$3^{62} + 3^{60} = 10(3^{y+1})$$

which can be rewritten as

$$3^{60}(3^2 + 1) = 10(3^{y+1})$$

or

$$3^{60}(10) = 10(3^{y+1})$$

or

$$3^{60} = 3^{y+1}$$

Hence it should be obvious that $y+1 = 60$ and $y = 59$.

Answer: 59 or $y = 59$

If the surface area of a cube is 0.54 square meters, what is the volume of the cube? Express your answer in cubic meters.

Clue: Since a cube has 6 sides, the area of one side of the cube is 0.09 square meters.

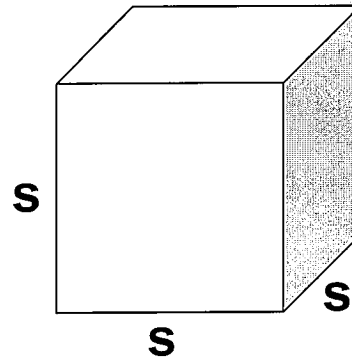
Round 4 Solution:

Following the clue, the area of one side of the cube is 0.09 square meters, which equals the square of its edge length (s). That is,

$$s^2 = 0.09 \text{ m}^2$$

Or

$$s = 0.3 \text{ m}$$



And its volume (V) is

$$V = s^3 = 0.3^3 \text{ m}^3 = 0.027 \text{ m}^3$$

Answer: 0.027 (m³)

The average (arithmetic mean) age of Matt, Mike, and Mary is 16, and the average age of Matt, Mike, and Moe is 20. If Moe is twice as old as Mary, what is the average of the ages of Mike and Matt?

Clue: Mary is less than 15 years old.

Round 5 Solution:

Since the average age of Mike, Matt, and Mary is 16, then the sum of their ages is 48 ($= 3 \times 16$). And it also follows that the sum of the ages of Mike, Matt, and Moe is 60 ($= 3 \times 20$). Letting the sum of the ages of Mike and Matt be X , and the age of Mary be Y , and Moe be $2Y$, the information above can be expressed algebraically as

$$X + Y = 48 \quad \text{and} \quad X + 2Y = 60$$

So, it's easy to determine Y to be 12, which is Mary's age, and X to be 36, which is the sum of the Mike's age and Matt's age. So their average age is 18. Notice that we did not need to determine their individual ages.

Answer: 18

What percent of $\frac{5}{6}$ is $\frac{1}{5}$?

Clue: The answer is an integer value between 20 and 30 percent.

Round 6 Solution:

First, we calculate the fraction that is represented by $\frac{1}{5}$ divided by $\frac{5}{6}$.

$$\frac{\frac{1}{5}}{\frac{5}{6}} = \frac{1}{5} \times \frac{6}{5} = \frac{6}{25}$$

And the percentage is simply

$$\frac{6}{25} \times 100\% = 24\%$$

Answer: 24 (%)

The towns of Marysville and Billsville are 300 miles apart. At 10 AM, a car leaving Marysville for Billsville travels at a rate of 55 miles per hour. Also at 10 AM, another car leaves Billsville for Marysville and travels at a rate of 50 miles per hour. When the two cars meet, how far away will they be from Marysville? Round your answer to the nearest mile.

Clue: When they meet, the cars are less than 160 miles from Marysville.

Round 7 Solution:

The first car leaving Marysville moves at a speed of 55 miles per hour and travels a distance D_1 . Similarly, the second care moves at a speed of 50 miles per hour and travels a distance D_2 . The cars meet at time T . That is

$$D_1 = 55T \quad \text{and} \quad D_2 = 50T$$

The distance traveled is 300. So,

$$D_1 + D_2 = 300.$$

Or

$$55T + 50T = 105T = 300$$

Hence, we can determine the time the cars meet,

$$T = 300/105 = 20/7 \text{ hours.}$$

Their distance from Marysville at this time can be computed from

$$D_1 = 55T = 55(20)/7 = 157.142 \text{ miles}$$

Which rounds to 157 miles.

Answer: 157 (miles)

A certain 7-digit number is expressed as $9,9AB,C20$, where each letter represents a single digit. If it is known that the number is the product of two consecutive even integers, determine the sum $A+B+C$.

Clue: $A = 9$.

Round 8 Solution:

The number represented by 9,9AB,C20 can be no smaller than 9,900,020 (which is A=B=C=0.) and no larger than 9,999,920 (which is A=B=C=9). Since consecutive integers are close in their values, determining the square root of each number will provide an estimate.

$$\sqrt{9,900,020} \cong 3146$$

and

$$\sqrt{9,999,920} \cong 3162$$

It follows that the pair of integers that represent the solution is one of the following: 3146 & 3148, or 3148 & 3150, or 3152 & 3154, . . . , 3160 & 3162.

A little trial and error leads to the following solution:

$$3160 \times 3162 = 9,991,920$$

which gives A = 9, B = 1, C = 9, and A+B+C = 19.

Answer: 19 or A+B+C = 19

Calculate the sum

$$1 + \left(1 - \frac{1}{2}\right)^2 + \left(1 - \frac{2}{3}\right)^3 + \left(1 - \frac{3}{4}\right)^4 + \dots + \left(1 - \frac{99}{100}\right)^{100}$$

And express your answer as a decimal to the nearest hundredth.

Round 9 Solution:

The given sum can be restated as

$$1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^3 + \left(\frac{1}{4}\right)^4 + \dots + \left(\frac{1}{100}\right)^{100}$$

As each term is evaluated as a decimal, notice that the values of higher terms quickly diminish. The values of the 4th, 5th, and 6th terms are 0.003906, 0.00032, and 0.00002143, respectively. If only accuracy to the nearest hundredth is required, then it should be clear that all terms after the 4th can be neglected, which is indicated in the table below,

Value of 1 st term =1.0	Sum of 1 st term = 1.0
Value of 2 nd term =0.25	Sum of first 2 terms =1.25
Value of 3 rd term =0.03704	Sum of first 3 terms =1.28704
Value of 4 th term =0.00391	Sum of first 4 terms =1.29095
Value of 5 th term =0.00032	Sum of first 5 terms =1.29127
Value of 6 th term =0.00002	Sum of first 6 terms =1.29129

Hence, to the nearest hundredth, the sum is 1.29.

Answer: 1.29