

2003-2004 CalcuSolve Bowl

Grades 7 & 8

Allegheny Intermediate Unit

Pittsburgh, Pennsylvania

Pipe A alone can drain a certain tank in x hours, and pipe B alone can drain the tank twice as fast as pipe A. If both pipes are opened for 3 hours, what fractional part of the tank is drained? Assume that $x > 9$ and express your answer in simplest terms of x .

Round 1 Solution:

In one hour, pipe A alone can drain a $\frac{1}{x}$ fractional part, of the tank. Likewise, pipe B alone can drain twice as much as pipe A, which is a $\frac{2}{x}$ fractional part of the tank. Together, in one hour, both pipes can drain a fractional part of the tank represented by

$$\frac{1}{x} + \frac{2}{x} = \frac{3}{x}$$

So, in 3 hours, the fractional part of the tank that is drained is simply

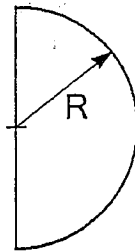
$$3 \times \frac{3}{x} = \frac{9}{x}$$

Answer: $\frac{9}{x}$

The radius of a semicircle is increased by 6 meters, and the resulting semicircle has a perimeter of 200 meters. What is the number of meters in the perimeter of the original semicircle? Express your answer to the nearest whole meter.

Clue: When its radius is increased by 6 meters, the resulting radius is 38.90 meters.

Round 2 Solution:



Referring to the figure, a semicircle of radius R has a perimeter that can be expressed as

$$P = 2R + 1/2(2\pi R) = 2R + \pi R = (2 + \pi)R$$

The resulting semicircle has a perimeter $P = 200$ meters. Solving the above equation for R gives

$$R = 200/(2 + \pi) = 38.90 \text{ meters}$$

The radius of the original circle is $38.90 - 6 = 32.90$ meters, from which we can calculate the original perimeter

$$P = (2 + \pi)32.90 = 169.25 \text{ meters}$$

Or, as a whole number of meters,

$$P = 169 \text{ meters}$$

Answer: 169 (meters)

There is a certain 2-digit number that has the following property. Written as a 3-digit number with a 9 before it, it is 9 more than the same number written as 3-digit number with a 9 after it. Find the 2-digit number.

Clue: The number is greater than 85.

Round 3 Solution:

Let X represent the sought-after 2-digit number. With a 9 in front of it, its value as a 3-digit number can be expressed as

$$900 + X$$

And with a 9 after it, its value can be expressed as

$$10X + 9$$

It's also given that the former is 9 more than the latter; that is

$$900 + X - (10X + 9) = 9$$

Solving for X gives

$$9X = 882$$

Or

$$X = 98$$

Answer: 98

When N fair coins are tossed, the probability of tossing all heads or all tails is $\frac{1}{64}$. Find N .

Clue: If $N = 3$, the probability of tossing all heads or all tail is $\frac{1}{4}$.

Round 4 Solution:

It is possible to solve this problem by a “guess and check” approach; that is, guess a value for N, work out all the permutations of heads and tails, work out the probability of getting all heads or all tails, and check that answer against the required probability of $1/64$. This problem can also be solved by recognizing the pattern that it expresses.

For example, for $N = 1$ there is $1/1$ (100%) probability that all heads or all tails result for a toss of 1 coin.

For $N = 2$, the permutations are

H-H, H-T, T-H, T-T

And there is a $1/2$ probability of tossing all heads or all tails.

In a similar manner, there will be a $1/4$ probability for $N=3$; a $1/8$ probability for $N=4$; a $1/16$ probability for $N=5$; and so on. Thus, there will be a $1/64$ probability for $N=7$

Answer: 7 or $N = 7$

At a Big Banana fruit stand, apples sell for 75 cents each and oranges for 50 cents each. If you buy 40 pieces of fruit (apples and oranges) for \$20.25, how many oranges did you buy?

Clue: More than 34 oranges were purchased.

Round 5 Solution:

Let X be the number of apples that were bought, and Y be the number of oranges that were bought. The problem statements becomes

$$X + Y = 40$$

and

$$75X + 50Y = 2025 \text{ cents } (= \$20.25)$$

Solving for X and Y , leads to $X = 1$ and $Y = 39$

A “guess and check” approach can also be used to get this result.

Answer: 39 (oranges)

Given that $x^n x^m = 1$, and $x > 1$, determine the value of $m + n$.

Clue: $m + n < 1$

Round 6 Solution:

The given equations is

$$x^n x^m = 1$$

This can be rearranged as

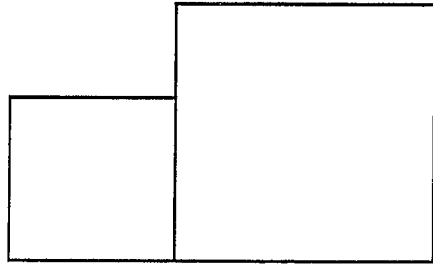
$$x^{n+m} = 1$$

It is also given that $x > 1$. This clue preclude $x=1$ or $x=-1$, which would permit many values for $m+n$. Thus, the only solution is $m+n = 0$, since

$$x^0 = 1$$

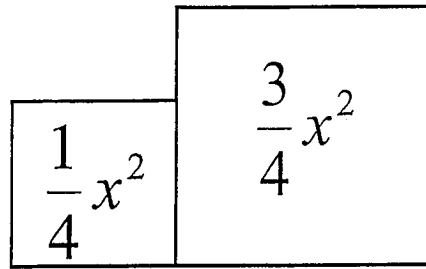
Answer: 0 or $m+n=0$

The area of the figure below is x^2 . If the smaller square has one-third the area of the larger square, what is the perimeter of the smaller square? Express your answer in terms of x .



Clue: The area of the larger square is $\frac{3}{4}x^2$.

Round 7 Solution:



Since the total area is x^2 , and the smaller square has one-third the area of the larger, then each square's area must be distributed as indicated above.

It follows that the edge length (s) of the smaller square is simply

$$s = \sqrt{\frac{1}{4}x^2} = \frac{1}{2}x$$

And its perimeter (p) is

$$p = 4s = 2x$$

Answer: $2x$

Determine the value of the following:

$$10 + 20 + 30 + \dots + 330 - 5 - 15 - 25 - \dots - 335$$

**Clue: It might be useful to rearrange the above
as**

$$10 - 5 + 20 - 15 + 30 - 25 \dots$$

Round 8 Solution:

From the clue, we have

$$10 - 5 + 20 - 15 + 30 - 25 \dots - 225 + 330 - 335$$

This can be written as

$$(10-5) + (20-15) + (30-25) \dots + (330-225) - 335$$

And simplified as

$$5 + 5 + 5 + \dots + 5 - 335$$

After a little thinking, we can see that there are 33 “fives,” and the final value can be determined as

$$33(5) - 335 = -170$$

Answer: -170

This is a non-calculator problem!

PUT YOUR CALCULATORS ASIDE.

Compute:

$$\frac{987,654,321}{(987,654,322)(987,654,324) - 987,654,323^2}$$

Round 9 Solution:

There are several ways to solve this problem, including brute force arithmetic. However, it's easy to make a mistake with a time-consuming, brute force computation. The problem can be greatly simplified by rewriting the denominator in terms of 987,654,323, which becomes

$$\frac{987,654,321}{(987,654,323 - 1)(987,654,323 + 1) - 987,654,323^2}$$

Or

$$\frac{987,654,321}{(987,654,323^2 - 1^2) - 987,654,323^2}$$

Or

$$\frac{987,654,321}{-1} = -987,654,321$$

Answer: $-987,654,321$

