

**Welcome to the
2012 Calcu-Solve Competition!
Northwest Pennsylvania Division**





We hope you have a challenging and successful day!
While we are waiting for all the teams to arrive, please:

1. Put your coats and lunches in an area where your team sponsors can keep an eye on them. The only things you need to take to your team's table are your calculators, and sharpened pencils (if you brought any.) Sit at the table with your team number on it.
2. **Make sure your team sponsor has completed the registration/scoring card that is in the folder he/she received when you arrived. This card should be filled out completely and given to the Registration Desk.**
3. Make a nametag for everyone in your group. Include your first and last names, school district, school name, and team number on the tag. Please wear the nametag during the entire competition.
4. Read over the information in the folder with your team sponsor. The rules and scoring procedures are explained. We will review these briefly just before the competition begins.
5. **Each person on your team should take one stapled packet of individual answer sheets from the team folder and fill in your full name and team number on EVERY sheet. Print neatly and accurately! Your team number is displayed on the sign at your table and on your team folder.**
6. Begin to practice for the competition by working on the Warm-Up Questions that are in your folder. We will go over the answers to these questions just before we begin the actual competition questions.
7. If you need help or further direction, please find a Student Assistant in a blue shirt or see Dr. Mr. Bancroft or Dr. Mrs. Bancroft.

Relax, Have Fun, and Good Luck!

Dr. Erin E. Bancroft



- Program Director
- Assistant Professor at GCC



Dr. Eric D. Bancroft



- Assistant Director
- Assistant Professor at GCC





II-VI Foundation

“Programs supported by the II-VI Foundation are designed to create a stronger population of new engineering-, science-, and mathematics-educated individuals that will ultimately increase and improve the pool of engineers and scientists seeking to tackle the tough and ever more complicated technical problems facing our nation and the world.”



“Thank you” to...

- II-VI Foundation
- Roxann Williams
- Jeff Prokovich
- Randy Cole
- Constance Nichols



...and all of our GCC student volunteers!

- Anna O'Neil
- Michele Perrine
- Hannah Chapman
- Ben DeClerico
- Jessica French
- Tanner Grudda
- Ben Harrington
- Jocelyn Hinkle
- Hannah Liermann
- Shannon Montgomery
- Christa Moore
- Josh Patterson
- Robin Park
- Emma Polaski
- Hannah Seaquist
- Ellie Stoffer
- Johanna Suffern
- Nathan Woodroof

Warm-Up Answers

- 6 minutes;** $\frac{1/4}{3/4} = \frac{x}{18}$ so $x = \frac{1}{3} \times 18 = 6$.
- Ursula $\frac{\text{-----}}{3m}$ Alma $\frac{\text{-----}}{3m}$ Cathy $\frac{\text{-----}}{2m}$ Lani $\frac{\text{-----}}{2m}$ Isabel $\frac{\text{-----}}{2m}$ Betty
- 30;** $3600/2 = 1800$, $1800/3 = 600$, $600/4 = 150$, $150/5 = 30$
- 11 bicycles and 4 tricycles;** $11 \times 2 + 4 \times 3 = 22 + 12 = 34$ wheels
- 22 friends and 5 cars;** $3x + 7 = 4x + 2$, so solving for x we get $x = 5$ or 5 cars. Then there are 3 times 5 plus 7 = 22 people.
- 60;** $15/7$ has a remainder of 1, $30/7$ has a remainder of 2, $45/7$ has a remainder of 3, $60/7$ has a remainder of 4.
- 28;** $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$.
- 400;** $400 + .50 \times 400 + .25 \times 400 = 400 + 200 + 100 = \700 .
- 3.9 miles;** $\frac{1.56}{12} = \frac{x}{30}$ and solving gives $x = 30 \times \frac{1.56}{12} = 3.9$.



Schedule and Explanation of Scoring

- There will be eight Individual Questions - #s 2, 3, 4, 5 and 7, 8, 9, 10.
- You will be given 4 minutes to earn 5 points for a correct answer on each Individual Question, or you may wait for a clue, work an extra 3 minutes and earn 3 points for a correct answer on each Individual Question.
- There will be two Group Questions - #s 1 and 6. Your team of students will be given 7 minutes to earn 10 points for a correct answer on each Group Question.
- After Group Question # 1 and Individual Questions # 2, 3, 4, and 5 we will take a short break.
- After the break, we will complete Group Question # 6 and Individual Questions # 7, 8, 9, and 10.
- Following Individual Question # 10, we will break for lunch.
- If necessary, “tie-breakers” will take place during lunch.
- Final scores will be announced and awards will be presented after ties are broken.
- Estimated concluding time is 1:30 p.m.



Guidelines for Tie-Breaking Situations

Individual Tie-Breakers*

1. In the event of a tied individual score, a sudden death question will be given to those participants who are tied. If an answer is turned in and it is incorrect, the person may continue to work on the problem. The first person with a correct answer within a 5-minute time limit will be declared the winner. If at the end of 5 minutes, no one has submitted a correct answer ...
2. ...another sudden death question will be given and step # 1 will be repeated. This procedure will be followed until a winner is determined.

*These rules will be used to determine first, second, third, and tenth place individual winners.

Group Tie-Breakers**

Group tie-breakers will be handled in the same fashion as individual except that the entire group will participate.

**These rules will be used to determine first, second, and third place teams only.



Sample Problem

Two mice are racing around the edges of a square whose sides are 2 feet in length. They start at the same corner and both go in a clockwise direction. One mouse travels at a constant rate of 1 foot per second, and the second mouse travels at a constant rate of 2 feet per second. After 22 seconds, how far apart will the mice be from each other?



Sample Problem - Clue

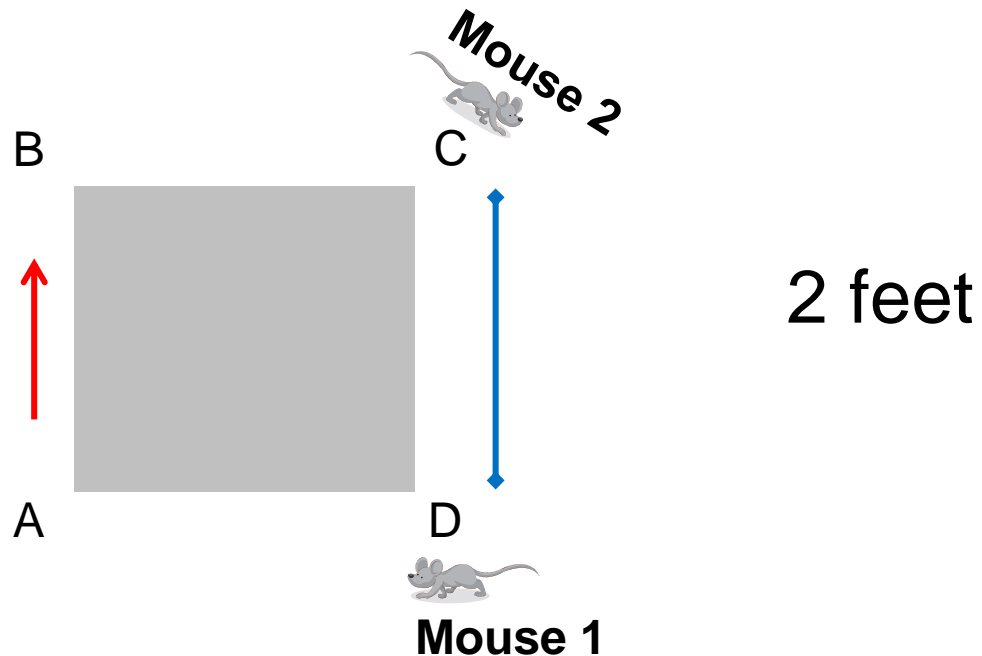
Two mice are racing around the edges of a square whose sides are 2 feet in length. They start at the same corner and both go in a clockwise direction. One mouse travels at a constant rate of 1 foot per second, and the second mouse travels at a constant rate of 2 feet per second. After 22 seconds, how far apart will the mice be from each other?

Clue: Draw a picture, see how they run.

Sample Problem - Solution

Two mice are racing around the edges of a square whose sides are 2 feet in length. They start at the same corner and both go in a clockwise direction. One mouse travels at a constant rate of 1 foot per second, and the second mouse travels at a constant rate of 2 feet per second. After 22 seconds, how far apart will the mice be from each other?

Solution:





Official Competition



Runners: Please pass out Group Question #1 face down and the green Group Answer Sheet #1.



Group Question #1

A bird collector wants to buy 20 birds and spend exactly \$20. Parakeets cost \$2 each, pigeons cost \$0.50 each, and love birds cost \$4 each. The collector wants at least one of each type of bird. How many pigeons does she buy?



Group Question #1 - Solution

A bird collector wants to buy 20 birds and spend exactly \$20. Parakeets cost \$2 each, pigeons cost \$0.50 each, and love birds cost \$4 each. The collector wants at least one of each type of bird. How many pigeons does she buy?

Solution:

Parakeets: \$2	Pigeons: \$.50	Love Birds: \$4	Total
6	12	2	\$26- too high
6	13	1	\$22.50 too high
5	14	1	\$21.00 too high
3	16	1	\$18.00 too low
2	16	2	\$20.00 just right!

16 pigeons





Runners: Please pass out Individual Question #1 face down.



Individual Question #1

Tracy asked Sarah to guess how many pairs of shoes she owned.

Tracy gave Sarah five hints:

“I have less than one hundred pairs of shoes.”

“The sum of the digits of the number of pairs of shoes I have is 10.”

“The number of pairs of shoes I have is not divisible by 4.”

“I do not have a prime number of pairs of shoes.”

“I can’t wear a different pair of shoes each week for an entire year.”

How many pairs of shoes does Tracy have?





Individual Question #1 - Clue

Tracy asked Sarah to guess how many pairs of shoes she owned. Tracy gave Sarah five hints:

"I have less than one hundred pairs of shoes."

"The sum of the digits of the number of pairs of shoes I have is 10."

"The number of pairs of shoes I have is not divisible by 4."

"I do not have a prime number of pairs of shoes."

"I can't wear a different pair of shoes each week for an entire year."

How many pairs of shoes does Tracy have?

Clue: A month has approximately four and a half weeks in it.





Individual Question #1 - Solution

Tracy asked Sarah to guess how many pairs of shoes she owned. Tracy gave Sarah five hints:

"I have less than one hundred pairs of shoes."

"The sum of the digits of the number of pairs of shoes I have is 10."

"The number of pairs of shoes I have is not divisible by 4."

"I do not have a prime number of pairs of shoes."

"I can't wear a different pair of shoes each week for an entire year."

How many pairs of shoes does Tracy have?

Solution:

19, 28, 37, 46, 55, 64, 73, 82, 91 – sum to 10 and less than 100

19, 37, 46, 55, 73, 82, 91 – not divisible by 4

46, 55, 82, 91 – not a prime number

46 – less than 52

46 pairs of shoes



Runners: Please pass out Individual Question #2
face down.



Individual Question #2

Alisha went shopping at three stores. At the first store she spent 10 percent of her money plus seven dollars. At the second store she spent 80 percent of her remaining money plus eight dollars. At the third store she spent 20 percent of her remaining money plus two dollars. When Alisha was done shopping she had two dollars left. How much money did she have when she started shopping?





Individual Question #2 - Clue

Alisha went shopping at three stores. At the first store she spent 10 percent of her money plus seven dollars. At the second store she spent 80 percent of her remaining money plus eight dollars. At the third store she spent 20 percent of her remaining money plus two dollars. When Alisha was done shopping she had two dollars left. How much money did she have when she started shopping?

Clue: If 30 percent is taken away, then I have 70 percent left.



Individual Question #2 - Solution

Alisha went shopping at three stores. At the first store she spent 10 percent of her money plus seven dollars. At the second store she spent 80 percent of her remaining money plus eight dollars. At the third store she spent 20 percent of her remaining money plus two dollars. When Alisha was done shopping she had two dollars left. How much money did she have when she started shopping?

Solution:

$$\text{Store Three: } \$2 + \$2 = \frac{\$4}{0.80} = \$5$$

$$\text{Store Two: } \$5 + \$8 = \frac{\$13}{0.20} = \$65$$

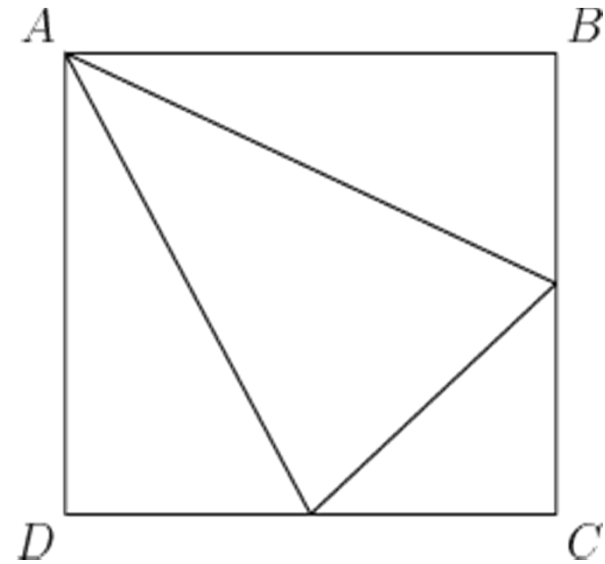
$$\text{Store One: } \$65 + \$7 = \frac{\$72}{0.90} = \mathbf{\$80}$$



Runners: Please pass out Individual Question #3 face down.

Individual Question #3

The area of the square ABCD is 64 in^2 . If point A and the midpoints of sides BC and CD are joined to form a triangle, what is the area of the triangle?

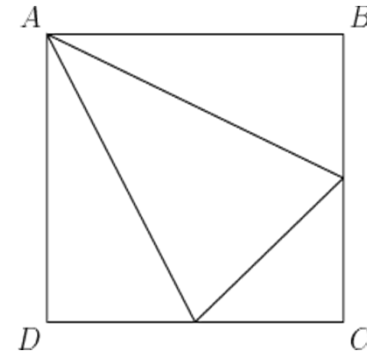




Individual Question #3 - Clue

The area of the square $ABCD$ is 64 in^2 .

If point A and the midpoints of sides BC and CD are joined to form a triangle, what is the area of the triangle?



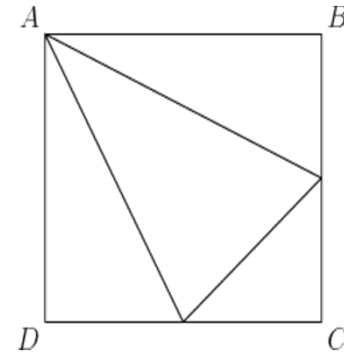
Clue: Two right triangles make a rectangle.



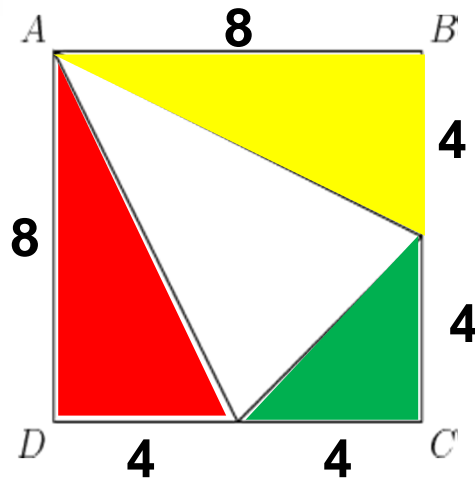
Individual Question #3 - Solution

The area of the square ABCD is 64 in^2 .

If point A and the midpoints of sides BC and CD are joined to form a triangle, what is the area of the triangle?



Solution:



Area of the square – the area of the red triangle – the area of the yellow triangle – the area of the green triangle

Area of a triangle is $\frac{1}{2} \times b \times h$

$$8 \times 8 - \frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 4 \times 4$$

$$= 64 - 16 - 16 - 8 = 24 \text{ in}^2$$



Runners: Please pass out Individual Question #4 face down.



Individual Question #4

Mrs. Brown gave a quiz in a math class of five students, and the scores were 14, 18, 20, 23 and 25. She entered the scores one at a time, in no particular order, into a spreadsheet that recalculated the average after each score was entered. Mrs. Brown noticed that after each score was entered, the average was always an integer. What was the average of the quiz scores before Mrs. Brown entered the last one?





Individual Question #4 - Clue

Mrs. Brown gave a quiz in a math class of five students, and the scores were 14, 18, 20, 23 and 25. She entered the scores one at a time, in no particular order, into a spreadsheet that recalculated the average after each score was entered. Mrs. Brown noticed that after each score was entered, the average was always an integer. What was the average of the quiz scores before Mrs. Brown entered the last one?

Clue: The average of 1 and 3 is 2.



Individual Question #4 - Solution

Mrs. Brown gave a quiz in a math class of five students, and the scores were 14, 18, 20, 23 and 25. She entered the scores one at a time, in no particular order, into a spreadsheet that recalculated the average after each score was entered. Mrs. Brown noticed that after each score was entered, the average was always an integer. What was the average of the quiz scores before Mrs. Brown entered the last one?

Solution:

20

$$14 + 18 + 20 + 23 = 75, \quad \frac{75}{4} = 18.75$$

$$14 + 18 + 20 + 25 = 77, \quad \frac{77}{4} = 19.25$$

$$14 + 18 + 23 + 25 = 80, \quad \frac{80}{4} = 20$$

$$14 + 20 + 23 + 25 = 82, \quad \frac{82}{4} = 20.5$$

$$18 + 20 + 23 + 25 = 86, \quad \frac{86}{4} = 21.5$$

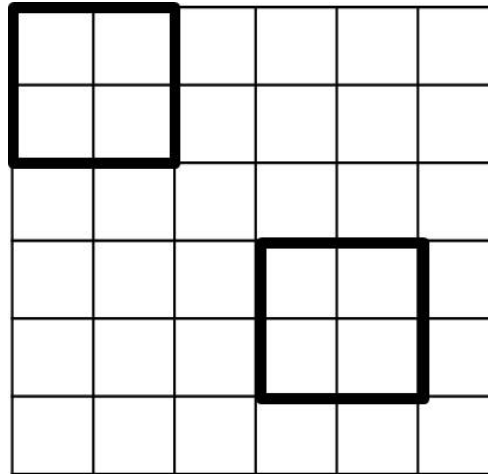


Runners: Please pass out Group Question #2 face down and the green Group Answer Sheet #2.

Group Question #2

How many squares of all possible sizes are there in a 6 x 6 grid made up of unit squares?

(Two such squares are shown in the 6 x 6 grid below.)

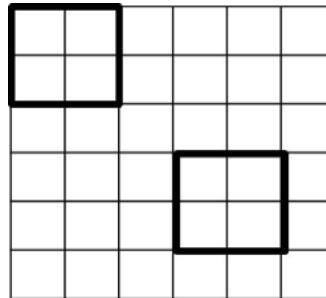




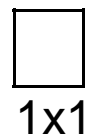
Group Question #2 - Solution

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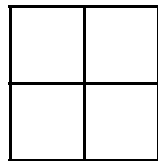


Solution: Look for a pattern.



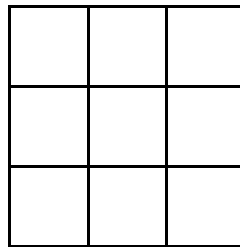
1x1

36



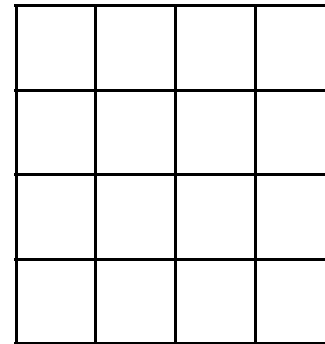
2x2

25



3x3

16



4x4

9

For 6 we'll get the first 6 perfect squares: $1 + 4 + 9 + 16 + 25 + 36 = 91$



Runners: Please pass out Individual Question #5 face down.



Individual Question #5

A palindrome is a number that remains the same when its digits are reversed, such as 83438. I'm thinking of a number that is a three digit palindrome. When you add 72 to my number, you get a four digit palindrome. What is my number?





Individual Question #5 - Clue

A palindrome is a number that remains the same when its digits are reversed, such as 83438. I'm thinking of a number that is a three digit palindrome. When you add 72 to my number, you get a four digit palindrome. What is my number?

Clue: "racecar" is a palindrome.





Individual Question #5 - Solution

A palindrome is a number that remains the same when its digits are reversed, such as 83438. I'm thinking of a number that is a three digit palindrome. When you add 72 to my number, you get a four digit palindrome. What is my number?

Solution:

We need a small four digit palindrome since it is only 72 away from a three digit palindrome. The smallest four digit palindromes are 1111 and 1001. When I subtract 72 from these numbers I get 1039 and 929. Since 929 is a three digit palindrome, it is my number!

929



Runners: Please pass out Individual Question #6 face down.



Individual Question #6

Sean drives his scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today he drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 35 minutes. How many minutes did he drive in the rain?





Individual Question #6 - Clue

Sean drives his scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today he drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 35 minutes. How many minutes did he drive in the rain?

Clue: If I drive 40 miles per hour that is $\frac{2}{3}$ mile per minute!



Individual Question #6 - Solution

Sean drives his scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today he drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 35 minutes. How many minutes did he drive in the rain?

Solution:

We need to use the equation distance = rate x time.

$\frac{1}{2}$ mile per minute - sun	$\frac{1}{3}$ mile per minute - rain	Total Miles
35 min.	0 min.	$\frac{1}{2} \times 35 = 17.5$
32	3	$\frac{1}{2} \times 32 + \frac{1}{3} \times 3 = 17$
30	5	$\frac{1}{2} \times 30 + \frac{1}{3} \times 5 = 16 \frac{2}{3}$
26	9	$\frac{1}{2} \times 26 + \frac{1}{3} \times 9 = 16$

9 minutes



Runners: Please pass out Individual Question #7
face down.



Individual Question #7

A store normally sells windows for \$100 each. This week the store is offering one free window with each purchase of four. Dave needs seven windows and Matt needs eight windows. To save money they decide to purchase the windows together. Each person will pay his share based on the number of windows he bought. Both men saved money by buying the windows together but one of the men saved more. How much money did the man who saved more, save?





Individual Question #7 - Clue

A store normally sells windows for \$100 each. This week the store is offering one free window with each purchase of four. Dave needs seven windows and Matt needs eight windows. To save money they decide to purchase the windows together. Each person will pay his share based on the number of windows he bought. Both men saved money by buying the windows together but one of the men saved more. How much money did the man who saved more, save?

Clue: 5 windows for the price of 4, what a deal!



Individual Question #7 - Clue

A store normally sells windows for \$100 each. This week the store is offering one free window with each purchase of four. Dave needs seven windows and Matt needs eight windows. To save money they decide to purchase the windows together. Each person will pay his share based on the number of windows he bought. Both men saved money by buying the windows together but one of the men saved more. How much money did the man who saved more, save?

Solution:

Dave by himself: 4 windows + 1 free + 2 more windows = \$600

Matt by himself: 4 windows + 1 free + 3 more windows = \$700

Dave and Matt together need 15 windows:

4 windows + 1 free + 4 windows + 1 free + 4 windows + 1 free = \$1200

Dave takes 7 of 15 windows so he pays $7/15 \times 1200 = \$560$

Matt takes 8 of 15 windows so he pays $8/15 \times 1200 = \$640$

Dave saves: $\$600 - \$560 = \$40$

Matt saves: $\$700 - \$640 = \$60$

\$60



Runners: Please pass out Individual Question #8 face down.



Individual Question #8

A special rubber ball is dropped from the top of a tower that is 243 feet high. Each time the ball hits the ground it bounces back only one-third as high as the distance it fell. The ball is caught when it bounces back to a high point of one foot. How many times does the ball hit the ground?





Individual Question #8 - Clue

A special rubber ball is dropped from the top of a tower that is 243 feet high. Each time the ball hits the ground it bounces back only one-third as high as the distance it fell. The ball is caught when it bounces back to a high point of one foot. How many times does the ball hit the ground?

Clue: The Eiffel tower is 1,063 feet tall!

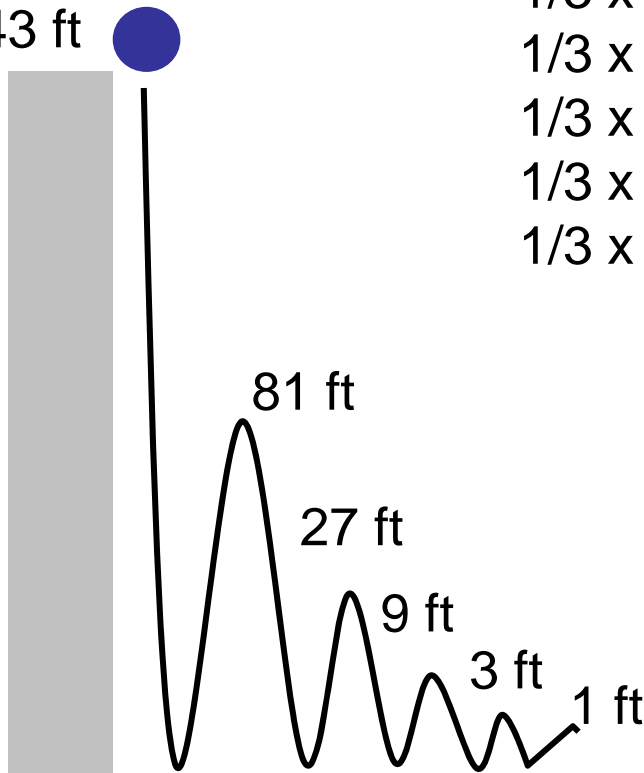


Individual Question #8 - Solution

A special rubber ball is dropped from the top of a tower that is 243 feet high. Each time the ball hits the ground it bounces back only one-third as high as the distance it fell. The ball is caught when it bounces back to a high point of one foot. How many times does the ball hit the ground?

Solution:

243 ft



$$1/3 \times 243 = 81$$

$$1/3 \times 81 = 27$$

$$1/3 \times 27 = 9$$

$$1/3 \times 9 = 3$$

$$1/3 \times 3 = 1$$

5



CALCU-SOLVE

MATHEMATICS COMPETITION

SPONSORED BY:

