


Welcome to the
2013 Calcu-Solve Competition!
Northwest Pennsylvania Division





We hope you have a challenging and successful day!
While we are waiting for all the teams to arrive, please:

1. Put your coats and lunches in an area where your team sponsors can keep an eye on them. The only things you need to take to your team's table are your calculators, and sharpened pencils (if you brought any.) Sit at the table with your team number on it.
2. **Make sure your team sponsor has completed the registration/scoring card that is in the folder he/she received when you arrived. This card should be filled out completely and given to the Registration Desk.**
3. Make a nametag for everyone in your group. Include your first and last names, school district, school name, and team number on the tag. Please wear the nametag during the entire competition.
4. Read over the information in the folder with your team sponsor. The rules and scoring procedures are explained. We will review these briefly just before the competition begins.
5. **Each person on your team should take one stapled packet of individual answer sheets from the team folder and fill in your full name and team number on EVERY sheet. Print neatly and accurately! Your team number is displayed on the sign at your table and on your team folder.**
6. Begin to practice for the competition by working on the Warm-Up Questions that are in your folder. We will go over the answers to these questions just before we begin the actual competition questions.
7. If you need help or further direction, please find a Student Assistant in a blue shirt or see Dr. Mr. Bancroft, Mrs. Potter, or Dr. Mrs. Bancroft.

Relax, Have Fun, and Good Luck!

Dr. Erin Bancroft



- Program Director
- Assistant Professor at GCC



Mrs. Sarah Potter



- Assistant Director
- Adjunct Professor at GCC



Dr. Eric Bancroft



- Assistant Director
- Assistant Professor at GCC





II-VI Foundation

“Programs supported by the II-VI Foundation are designed to create a stronger population of new engineering-, science-, and mathematics-educated individuals that will ultimately increase and improve the pool of engineers and scientists seeking to tackle the tough and ever more complicated technical problems facing our nation and the world.”



“Thank you” to...

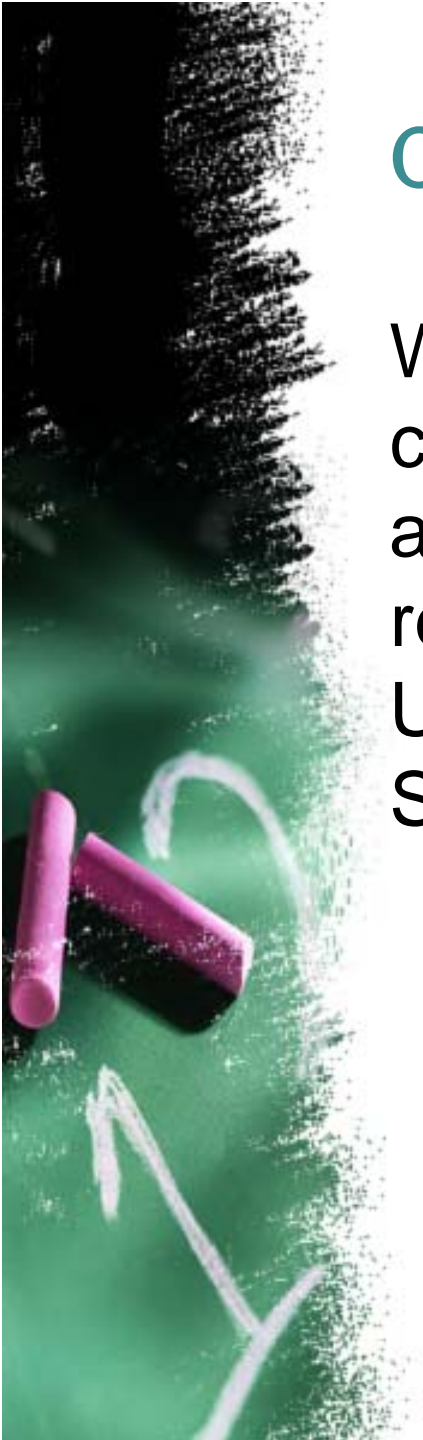
- II-VI Foundation
- Roxann Williams

...and all of our GCC student volunteers!

- Anna O’Neil
- Michele Perrine
- Kelsey Beevers
- Jake Benbow
- Ben DeClerico
- John Hughes
- Kyle Jones
- Corrie McNulty
- Emily Owen
- Robin Park
- Arianna Sternadori
- Lydia Staats
- Ellie Stoffer

Calcu-Solve Super Bowl!

Winning teams and individuals from our competition will be invited to compete against the winners from two other regional competitions at Duquesne University on the Friday before the Super Bowl.





7th and 8th grade Calcu-Solve Competition

Next year we will be hosting a competition for 7th and 8th graders on

Thursday, November 6th

5th and 6th grade Calcu-Solve Competition

Will be held next year (2014) on

Friday, November 7th



Warm-Up Answers

- 1. 180 feet;** $6x + 2x = 240$ so $x = 30$ and the longest piece is $6 \times 30 = 180$.
- 2. 7;** $4 \times 5 \times 6 \times 7 = 840$.
- 3. 100;** The first term is $4 = 4 \times 1$, the second term is $8 = 4 \times 2$, the third term is $12 = 4 \times 3$, so the 25th term is $4 \times 25 = 100$.
- 4. 60;** $12 = 2 \times 2 \times 3$ and $15 = 3 \times 5$ so our number must be at least $2 \times 2 \times 3 \times 5 = 60$. Let's check the factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60 which is a total of 12 factors.
- 5. 30 pints;** $3\frac{3}{4} \times 4 = 15$ quarts and $15 \times 2 = 30$ pints.
- 6. 13 green marbles;** Before we add any the probability of picking a green marble is $\frac{7}{8+7+2} = \frac{7}{17}$. If we add 13 green marbles the probability is $\frac{20}{8+20+2} = \frac{20}{30} = \frac{2}{3}$.
- 7. \$510;** Calculating the first discount (60% off means you pay for 40%): $\$1500 \times 40\% = \600 and then the coupon (15% off means you pay for 85%) $\$600 \times 85\% = \510 .
- 8. $\frac{19}{27}$;** Derek has $\frac{1}{3} = \frac{9}{27}$ of his pizza left, Jamal has $\frac{2}{9} = \frac{6}{27}$ of his pizza left and Margaret has $\frac{4}{27}$ of her pizza left. Adding them together we get $\frac{9}{27} + \frac{6}{27} + \frac{4}{27} = \frac{19}{27}$.
- 9. 11 turns;** First try: $46 = 9 \times 5 + 1$ which doesn't work. Next try: $46 = 8 \times 5 + 6 = 8 \times 5 + 3 \times 2$ which works for a total of $8 + 3 = 11$ turns.



Schedule and Explanation of Scoring

- There will be eight Individual Questions - #s 2, 3, 4, 5 and 7, 8, 9, 10.
- You will be given 4 minutes to earn 5 points for a correct answer on each Individual Question, or you may wait for a clue, work an extra 3 minutes and earn 3 points for a correct answer on each Individual Question.
- There will be two Group Questions - #s 1 and 6. Your team of students will be given 7 minutes to earn 10 points for a correct answer on each Group Question.
- After Group Question # 1 and Individual Questions # 2, 3, 4, and 5 we will take a short break.
- After the break, we will complete Group Question # 6 and Individual Questions # 7, 8, 9, and 10.
- Following Individual Question # 10, we will break for lunch.
- If necessary, “tie-breakers” will take place during lunch.
- Final scores will be announced and awards will be presented after ties are broken.
- Estimated concluding time is 1:30 p.m.



Guidelines for Tie-Breaking Situations

Individual Tie-Breakers*

1. In the event of a tied individual score, a sudden death question will be given to those participants who are tied. If an answer is turned in and it is incorrect, the person may continue to work on the problem. The first person with a correct answer within a 5-minute time limit will be declared the winner. If at the end of 5 minutes, no one has submitted a correct answer ...
2. ...another sudden death question will be given and step # 1 will be repeated. This procedure will be followed until a winner is determined.

*These rules will be used to determine first, second, third, and tenth place individual winners.

Group Tie-Breakers**

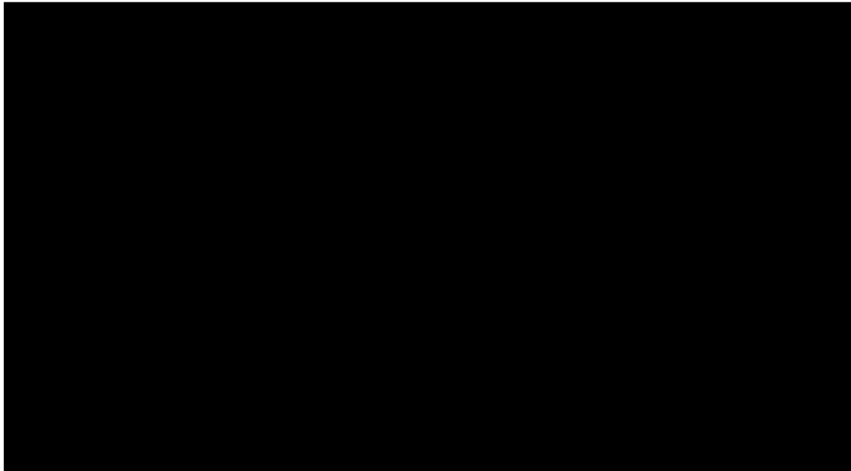
Group tie-breakers will be handled in the same fashion as individual except that the entire group will participate.

**These rules will be used to determine first, second, and third place teams only.



Sample Problem

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?






Sample Problem - Clue

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Clue: A $6 \times 6 \times 6$ hole is *not* twice as big as a $3 \times 3 \times 3$ hole.



Sample Problem - Solution

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Solution: 16 hours.

The first hole has volume $3 \times 3 \times 3 = 27$ cubic units. The new hole has volume $6 \times 6 \times 6 = 216$ cubic units. $\frac{216}{27} = 8$, so 8 smaller holes would fit in the new hole, which means it will take $8 \times 2 = 16$ hours to dig the new hole.





Official Competition



Runners: Please pass out Group Question #1 face down and the green Group Answer Sheet #1.

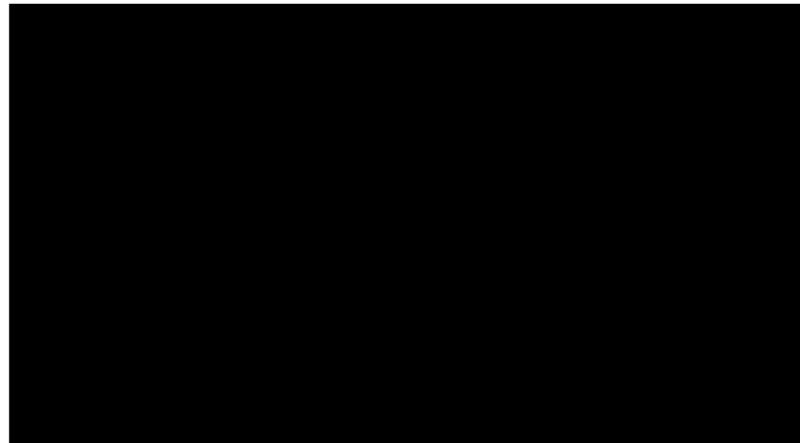




Group Question #1

Lydia and Alan wanted to make a rectangular box with integer (whole number) length sides out of plywood. At the lumber yard, they realized that they had forgotten the dimensions of the box. Lydia remembered that one of the sides had an area of 40 square centimeters.

Alan remembered that the other sides had areas of 150 and 60 square centimeters. What is the length of the longest side of the box?



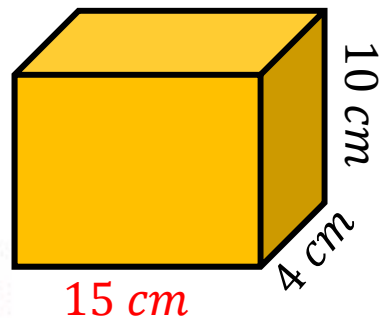


Group Question #1 - Solution

Lydia and Alan wanted to make a rectangular box with integer length sides out of plywood. At the lumber yard, they realized that they had forgotten the dimensions of the box. Lydia remembered that one of the sides had an area of 40 square centimeters. Alan remembered that the other sides had areas of 150 and 60 square centimeters. What is the length of the longest side of the box?

Solution:

Look at common factors among the areas, then guess and check to find the combination that works. You can also use an algebraic approach: If the dimensions are a by b by c , then $ab = 40$, $bc = 60$, and $ac = 150$. Multiply the first two together to get $ab^2c = 2400$. Dividing both sides by ac gives $b^2 = 2400/ac = 2400/150 = 16$. Then $b = 4$ and we can use the other equations to find that $a = 10$ and $c = 15$.



So the length of the longest side is **15 cm**.




Runners: Please pass out Individual Question #1 face down.





Individual Question #1

A recipe that makes 36 cookies calls for $2\frac{1}{2}$ cups of flour. A 5 pound bag of flour contains 20 cups. How many pounds of flour are needed to make 432 cookies? Express your answer as a decimal to the nearest hundredth.







Individual Question #1 - Clue

A recipe that makes 36 cookies calls for $2\frac{1}{2}$ cups of flour. A 5 pound bag of flour contains 20 cups. How many pounds of flour are needed to make 432 cookies? Express your answer as a decimal to the nearest hundredth.

Clue: Don't forget to convert to pounds!





Individual Question #1 - Solution

A recipe that makes 36 cookies calls for $2\frac{1}{2}$ cups of flour. A 5 pound bag of flour contains 20 cups. How many pounds of flour are needed to make 432 cookies? Express your answer as a decimal to the nearest hundredth.

Solution:

To make 432 cookies we must make $\frac{432}{36} = 12$ batches of cookies. So we need a total of $12 \times 2\frac{1}{2} = 30$ cups of flour. Since each bag contains 20 cups, we need $\frac{30}{20} = 1\frac{1}{2}$ bags of flour which is $1\frac{1}{2} \text{ bags} \times 5 \text{ lbs/bag} = 7.50 \text{ lbs}$ of flour.



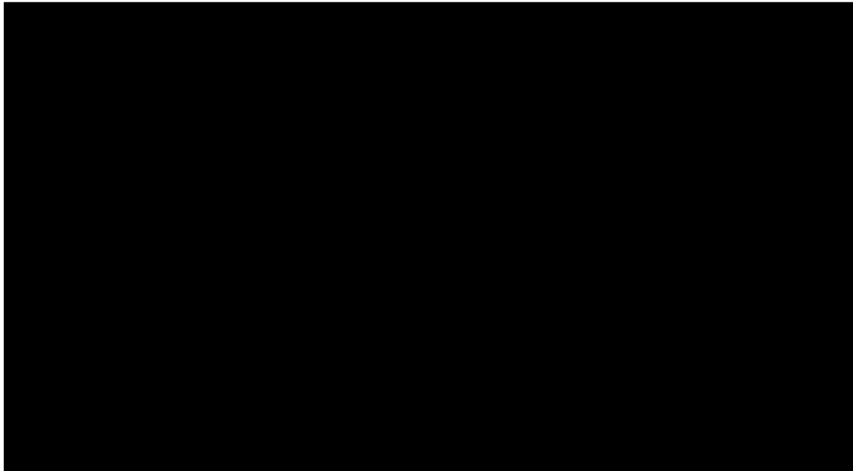
Runners: Please pass out Individual Question #2 face down.





Individual Question #2

At the Letter Store, each vowel sells for a different price, but all consonants are free. The word “ordered” sells for \$10, “mathematics” sells for \$20, “decimal” sells for \$15, “eighteen” sells for \$16, and “number” sells for \$7. What is the dollar cost of the word “CalcuSolve”?







Individual Question #2 - Clue

At the Letter Store, each vowel sells for a different price, but all consonants are free. The word “ordered” sells for \$10, “mathematics” sells for \$20, “decimal” sells for \$15, “eighteen” sells for \$16, and “number” sells for \$7. What is the dollar cost of the word “CalcuSolve”?

Clue: What is the difference between buying “decimal” and “mathematics”?







Individual Question #2 - Solution

At the Letter Store, each vowel sells for a different price, but all consonants are free. The word “ordered” sells for \$10, “mathematics” sells for \$20, “decimal” sells for \$15, “eighteen” sells for \$16, and “number” sells for \$7. What is the dollar cost of the word “CalcuSolve”?

Solution:

We need one a , one u , one o , and one e . We know that:

$$o + e + e = 10$$

$$a + a + e + i = 20$$

$$a + e + i = 15$$

$$e + e + e + i = 16$$

$$u + e = 7$$

The second and third equations tell us that one a costs \$5, so from the third equation we find that $e + i = 10$.

Then the fourth equation tells us that $e + e = 6$, so an e costs \$3. Then an o costs $\$10 - 2(\$3) = \$4$ and a u costs $\$7 - \$3 = \$4$. So one a , one u , one o , and one e will cost

$$\$5 + \$4 + \$4 + \$3 = \mathbf{\$16}$$



Runners: Please pass out Individual Question #3 face down.



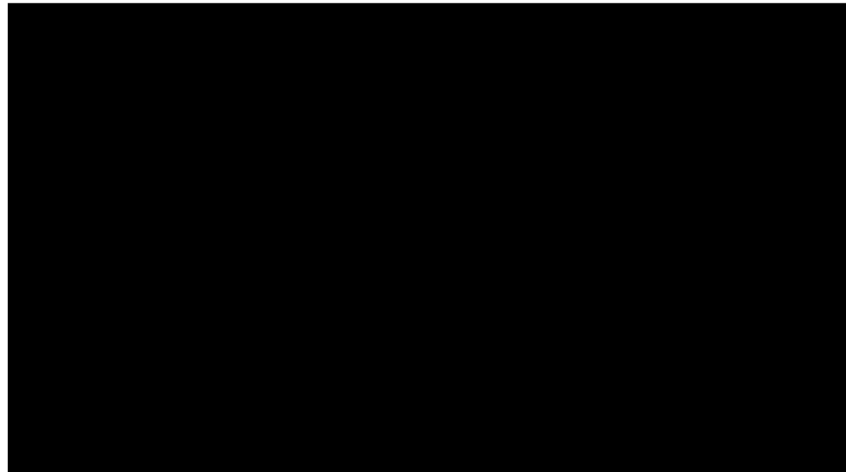


Individual Question #3

Five distinct points A, B, C, D and E lie on a line, but not necessarily in that order. We know that:

- E is the midpoint of segment AB ,
- D is the midpoint of segment AE ,
- Both C and E are the same distance from B ,
- The distance from D to B is 21 feet.

What is the length of segment DC ?







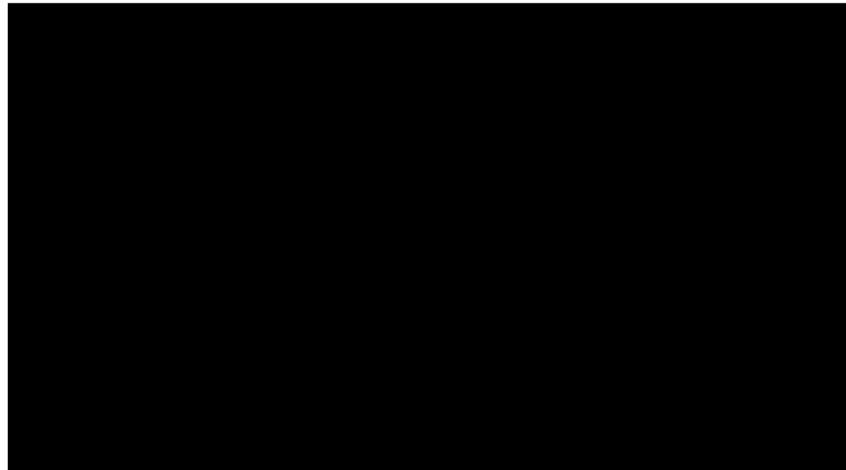
Individual Question #3 - Clue

Five distinct points A, B, C, D and E lie on a line, but not necessarily in that order. We know that:

- E is the midpoint of segment AB ,
- D is the midpoint of segment AE ,
- Both C and E are the same distance from B ,
- The distance from D to B is 21 feet.

What is the length of segment DC ?

Clue: Draw a picture and compare distances.





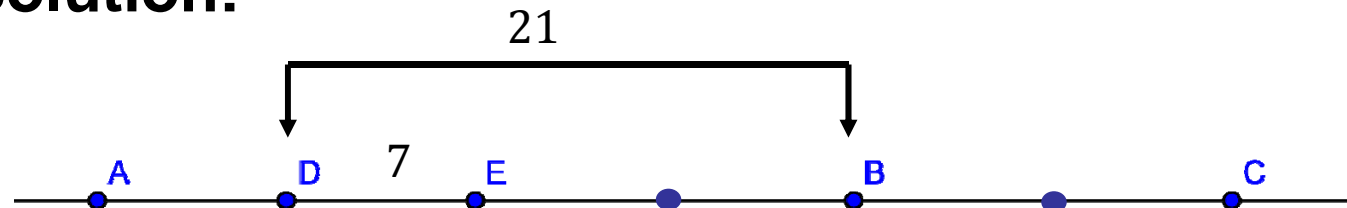
Individual Question #3 - Solution

Five distinct points A, B, C, D and E lie on a line, but not necessarily in that order. We know that:

- E is the midpoint of segment AB ,
- D is the midpoint of segment AE ,
- Both C and E are the same distance from B ,
- The distance from D to B is 21 feet.

What is the length of segment DC ?

Solution:



Based on the given information the distance between any two of the points we labeled on the graph is the same, so $DE = \frac{21}{3} = 7$ feet. Then since DC contains 5 segments we have $DC = 5 \times 7 = 35$ feet.

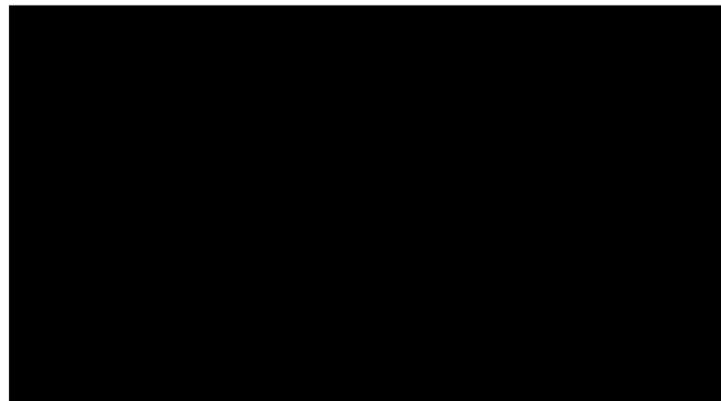
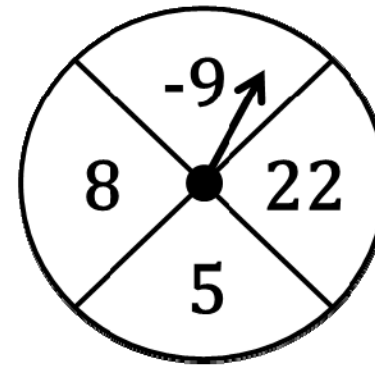
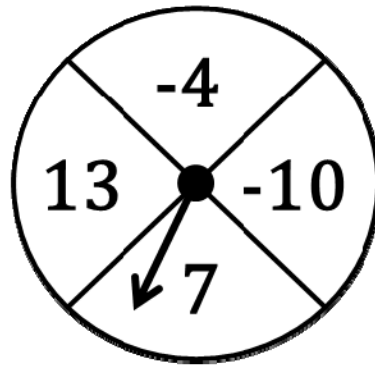


Runners: Please pass out Individual Question #4 face down.



Individual Question #4

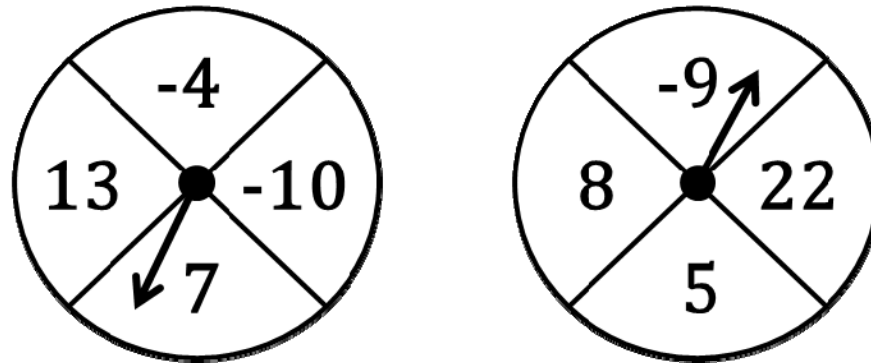
Two circles with spinners at their centers are divided into four equal regions as shown. When both spinners are spun, what is the probability that the product of the two values the spinners land on is negative? Express your answer as a fraction in lowest terms.



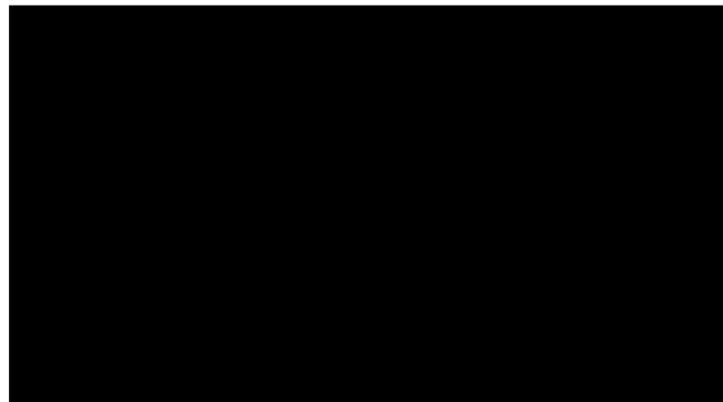


Individual Question #4 - Clue

Two circles with spinners at their centers are divided into four equal regions as shown. When both spinners are spun, what is the probability that the product of the two values the spinners land on is negative? Express your answer as a fraction in lowest terms.



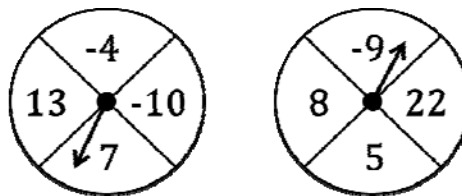
Clue: If I roll one die, the probability that I get a 5 is $\frac{1}{6}$.





Individual Question #4 - Solution

Two circles with spinners at their centers are divided into four equal regions as shown. When both spinners are spun, what is the probability that the product of the two values the spinners land on is negative? Express your answer as a fraction in lowest terms.



Solution:

There are 4 different numbers the left spinner could land on and 4 different numbers the right spinner could land on for a total of 16 different pairs (or 16 different products). For each number the left spinner lands on we can see which numbers the right spinner could land on so that the product is

negative:

-4	8, 5, 22	3 products	For a total of
13	-9	1 product	$3 + 1 + 1 + 3 = 8$ products
7	-9	1 product	that are negative. So the
-10	8, 5, 22	3 products	probability is $\frac{8}{16} = \frac{1}{2}$.

Snack Time





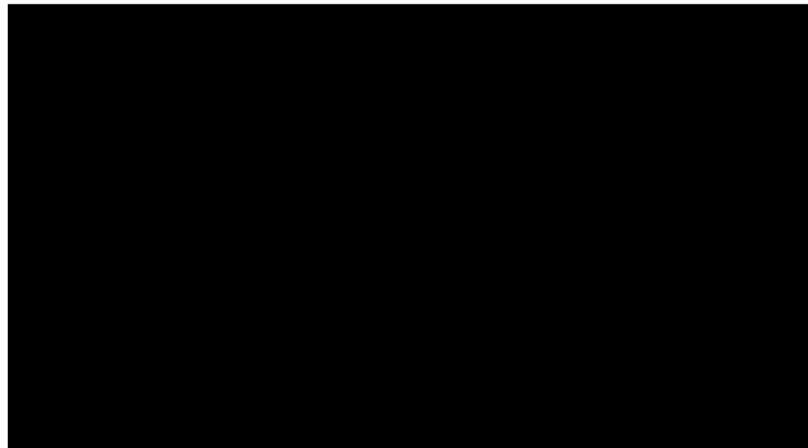
Runners: Please pass out Group Question #2
face down and the green Group Answer Sheet #2.





Group Question #2

On the first day, magic Mel used his wand to make the length of a 1 inch flower increase by $\frac{1}{2}$ so that the flower was now a longer length of $\frac{3}{2}$ inch. On the second day, he increased the flower's longer length by $\frac{1}{3}$; on the third day he increased the flower's new length by $\frac{1}{4}$; and so on. On what day of performing this trick will Magic Mel make the flower's length exactly 100 inches?





Group Question #2 - Solution

On the first day, magic Mel used his wand to make the length of a 1 inch flower increase by $\frac{1}{2}$ so that the flower was now a longer length of $\frac{3}{2}$ inch. On the second day, he increased the flower's longer length by $\frac{1}{3}$; on the third day he increased the flower's new length by $\frac{1}{4}$; and so on. On what day of performing this trick will Magic Mel make the flower's length exactly 100 inches?

Solution:

On the first day the length is $1 \times \frac{3}{2} = \frac{3}{2}$ inches.

On the second day the length is $\frac{3}{2} \times \frac{4}{3} = \frac{4}{2} = 2$ inches.

On the third day the length is $2 \times \frac{5}{4} = \frac{5}{2}$ inches.

On the fourth day the length is $\frac{5}{2} \times \frac{6}{5} = \frac{6}{2} = 3$ inches.

This pattern continues, so on the **198th day** the length is

$$\frac{198+2}{2} = \frac{200}{2} = 100 \text{ inches.}$$



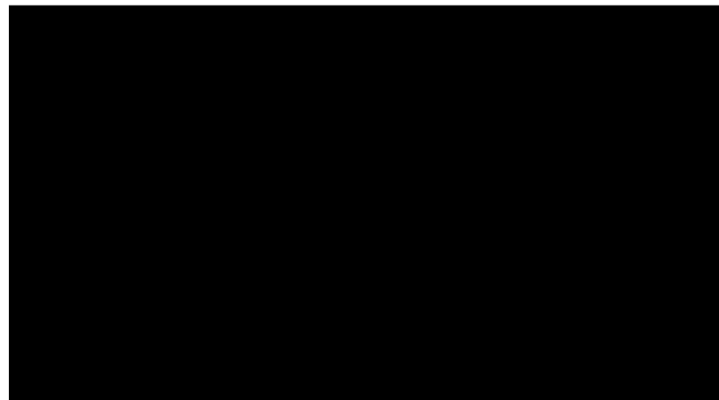
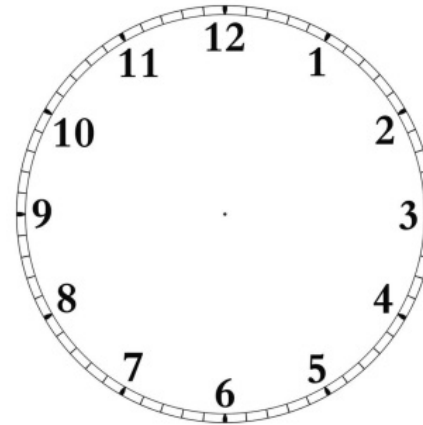


Runners: Please pass out Individual Question #5 face down.



Individual Question #5

How many different rectangles can be drawn using the hour marks on a clock's face as corners?

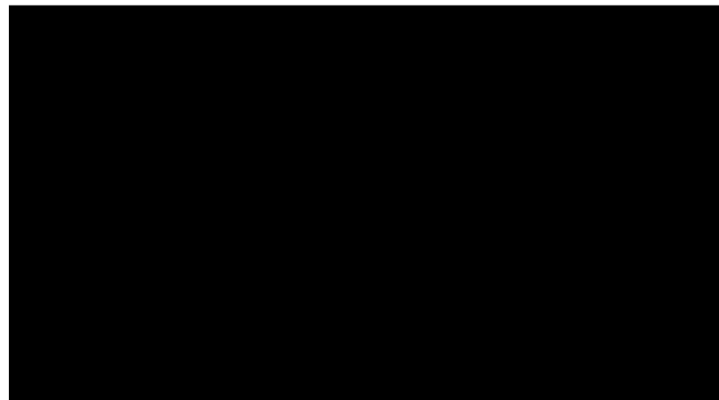
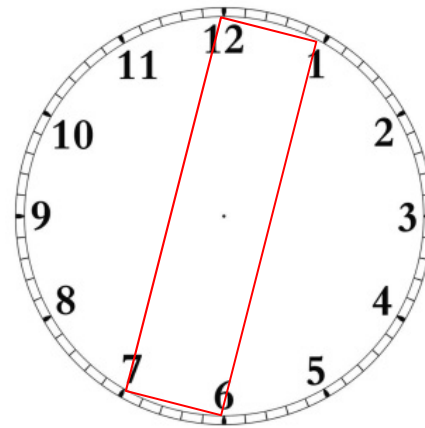




Individual Question #5 - Clue

How many different rectangles can be drawn using the hour marks on a clock's face as corners?

Clue: One of the rectangles looks like this:



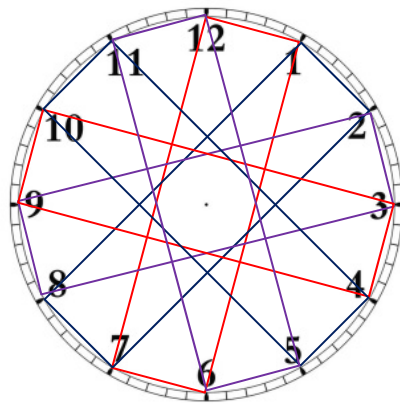


Individual Question #5 - Solution

How many different rectangles can be drawn using the hour marks on a clock's face as corners?

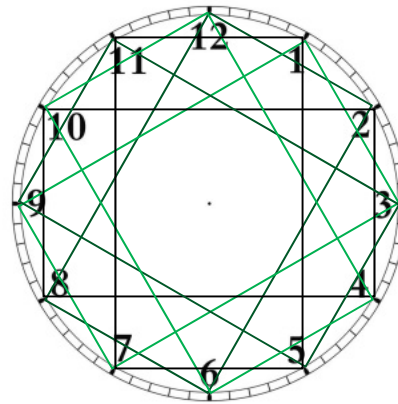
Solution:

There are



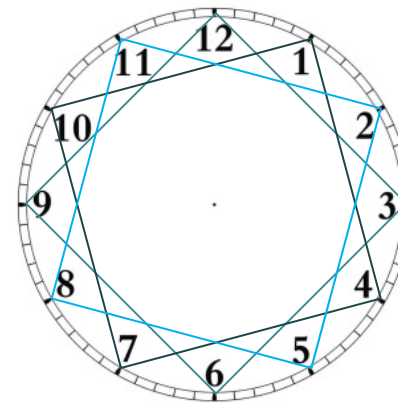
6

+



6

+



3

= 15 rectangles



Runners: Please pass out Individual Question #6 face down.



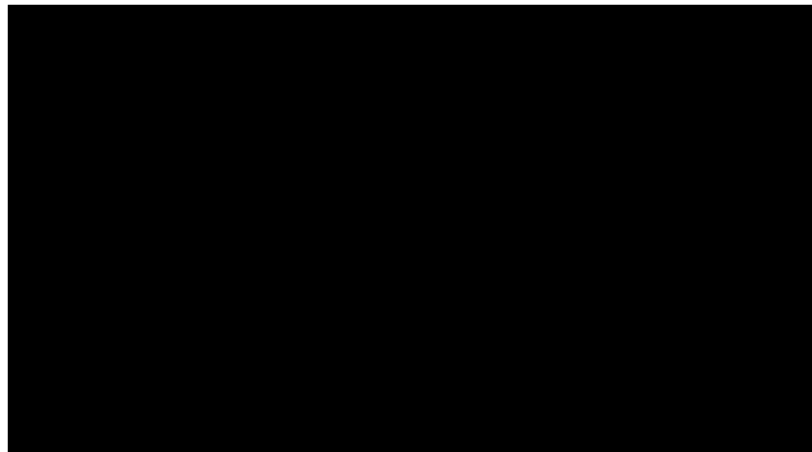


Individual Question #6

Theresa has created these rules for generating lists of whole numbers:

- If a number is 20 or less, double the number.
- If a number is more than 20, subtract 14 from it.

For example, if Theresa starts with 10, she gets the list 10, 20, 40, 26, 12, If the third number in Theresa's list is 24, what is the sum of the four distinct numbers that could have been the first number in the list?







Individual Question #6 - Clue

Theresa has created these rules for generating lists of whole numbers:

- If a number is 20 or less, double the number.
- If a number is more than 20, subtract 14 from it.

For example, if Theresa starts with 10, she gets the list 10, 20, 40, 26, 12,
If the third number in Theresa's list is 24, what is the sum of the four distinct numbers that could have been the first number in the list?

Clue: She could have gotten 24 by doubling a number and then subtracting 14 once.







Individual Question #6 - Solution

Theresa has created these rules for generating lists of whole numbers:

- *If a number is 20 or less, double the number.*
- *If a number is more than 20, subtract 14 from it.*

(For example, if Theresa starts with 10, she gets the list 10, 20, 40, 26, 12,) If the third number in Theresa's list is 24, what is the sum of the four distinct numbers that could have been the first number in the list?

Solution:

Theresa could have gotten to 24 by doubling both times, so we'll divide by two twice: $24 \rightarrow 12 \rightarrow 6$

She also could have gotten to 24 by subtracting 14 both times, so we'll add 14 twice: $24 \rightarrow 38 \rightarrow 52$

She also could have subtracted 14 and then doubled, so we'll divide by two and then add 14: $24 \rightarrow 12 \rightarrow 26$

She also could have doubled and then subtracted, so we'll add 14 and then divide by two: $24 \rightarrow 38 \rightarrow 19$

So the sum of four possible first numbers is

$$6 + 52 + 26 + 19 = 103$$



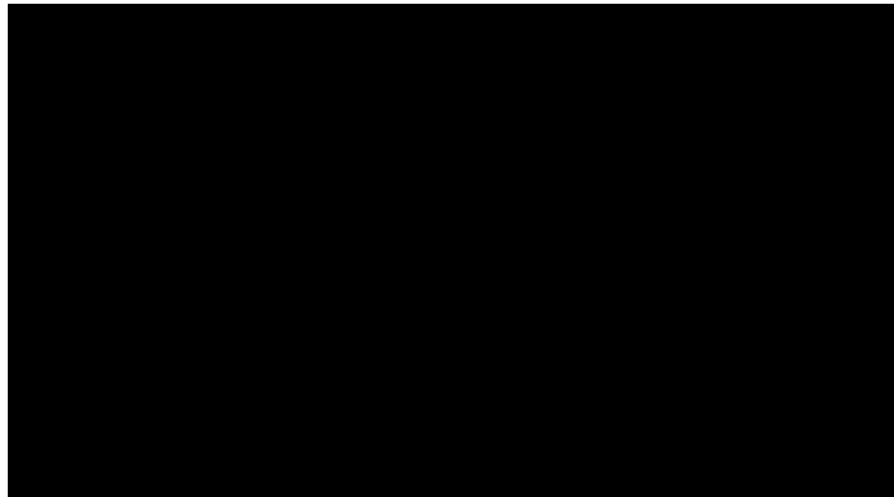
Runners: Please pass out Individual Question #7 face down.





Individual Question #7

Five real numbers are chosen and put in order from smallest to largest. The average of all five is 15. The average of the three middle numbers is only 13. What is the average of the largest and smallest numbers?







Individual Question #7 - Clue

Five real numbers are chosen and put in order from smallest to largest. The average of all five is 15. The average of the three middle numbers is only 13. What is the average of the largest and smallest numbers?

Clue: Start by finding the sum of all five numbers.







Individual Question #7 - Solution

Five real numbers are chosen and put in order from smallest to largest. The average of all five is 15. The average of the three middle numbers is only 13. What is the average of the largest and smallest numbers?

Solution:

The sum of all five numbers is $5 \cdot 15 = 75$ and the sum of the middle three numbers is $3 \cdot 13 = 39$. So the sum of just the largest and smallest numbers is $75 - 39 = 36$, and thus their average is

$$\frac{36}{2} = 18.$$



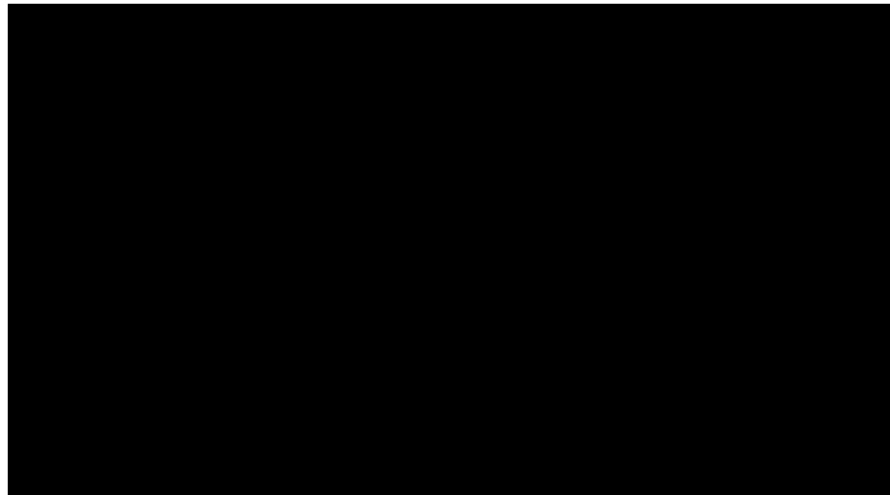
Runners: Please pass out Individual Question #8 face down.





Individual Question #8

Sam is feeding his 32 fish. Sam feeds his betta fish 3 pellets of food a day and he feeds his goldfish 5 pellets of food a day. Sam uses a total of 130 pellets of food a day. How many betta fish does Sam have?







Individual Question #8 - Clue

Sam is feeding his 32 fish. Sam feeds his betta fish 3 pellets of food a day and he feeds his goldfish 5 pellets of food a day. Sam uses a total of 130 pellets of food a day. How many betta fish does Sam have?

Clue: Every fish gets at least 3 pellets, but the goldfish get more...







Individual Question #8 - Solution

Sam is feeding his 32 fish. Sam feeds his betta fish 3 pellets of food a day and he feeds his goldfish 5 pellets of food a day. Sam uses a total of 130 pellets of food a day. How many betta fish does Sam have?

Solution:

Once Sam gives every fish 3 pellets he will have $130 - 32 \cdot 3 = 34$ pellets left to divide up among his goldfish. Each goldfish gets 2 more pellets, so there are $\frac{34}{2} = 17$ goldfish and $32 - 17 = 15$ betta fish.



CALCU-SOLVE

MATHEMATICS COMPETITION

SPONSORED BY:

