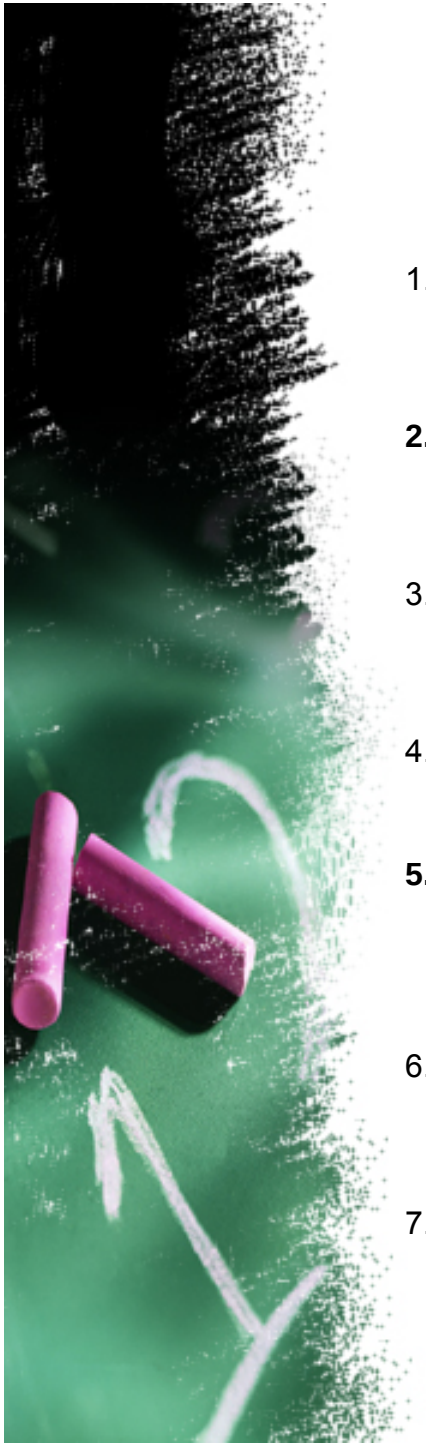
Welcome to the 2014 Calcu-Solve Competition!

Northwest Pennsylvania Division



We hope you have a challenging and successful day!
While we are waiting for all the teams to arrive, please:

1. Put your coats and lunches in an area where your team sponsors can keep an eye on them. The only things you need to take to your team's table are your calculators, and sharpened pencils (if you brought any.) Sit at the table with your team number on it.

2. **Make sure your team sponsor has completed the registration/scoring card that is in the folder he/she received when you arrived. This card should be filled out completely and given to the Registration Desk.**

3. Make a nametag for everyone in your group. Include your first and last names, school district, school name, and team number on the tag. Please wear the nametag during the entire competition.

4. Read over the information in the folder with your team sponsor. The rules and scoring procedures are explained. We will review these briefly just before the competition begins.

5. **Each person on your team should take one stapled packet of individual answer sheets from the team folder and fill in your full name and team number on EVERY sheet. Print neatly and accurately! Your team number is displayed on the sign at your table and on your team folder.**

6. Begin to practice for the competition by working on the Warm-Up Questions that are in your folder. We will go over the answers to these questions just before we begin the actual competition questions.

7. If you need help or further direction, please find a Student Assistant in a green shirt or see Mrs. Potter or Dr. Mrs. Bancroft.

Relax, Have Fun, and Good Luck!

Dr. Erin Bancroft

- Program Director
- Assistant Professor at GCC

Mrs. Sarah Potter

- Assistant Director
- Adjunct Professor at GCC

II-VI Foundation

"Programs supported by the II-VI Foundation are designed to create a stronger population of new engineering-, science-, and mathematics-educated individuals that will ultimately increase and improve the pool of engineers and scientists seeking to tackle the tough and ever more complicated technical problems facing our nation and the world."

“Thank you” to…

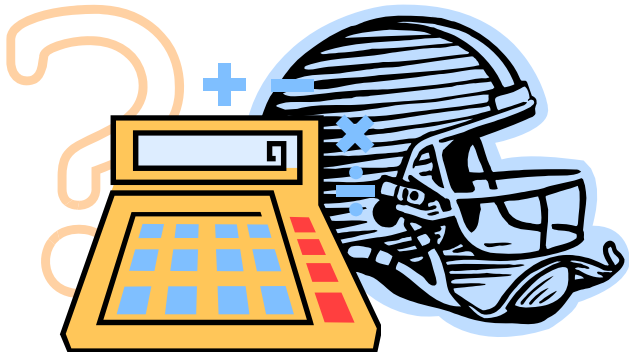
- II-VI Foundation
- Roxann Williams

…and all of our GCC student volunteers!

- Anna O’Neil
- Julia Berek
- Shelby Davenport
- Sarah Deemer
- Katy Gibson
- Kendra Helfrich
- Annie Laurie Holfelder
- Amanda Johnson
- Robin Lawson
- Hannah Liermann
- Kelly Scobee
- David Shang
- Abby Slater
- Bonnie Stahl
- Kailey Tuhacek
- Brittany Turner
- Hannah Wierenga
- Cindy Zohoranacky

Calcu-Solve Super Bowl!

Winning teams and individuals from our competition will be invited to compete against the winners from two other regional competitions at Duquesne University in February.



7th and 8th grade Calcu-Solve Competition

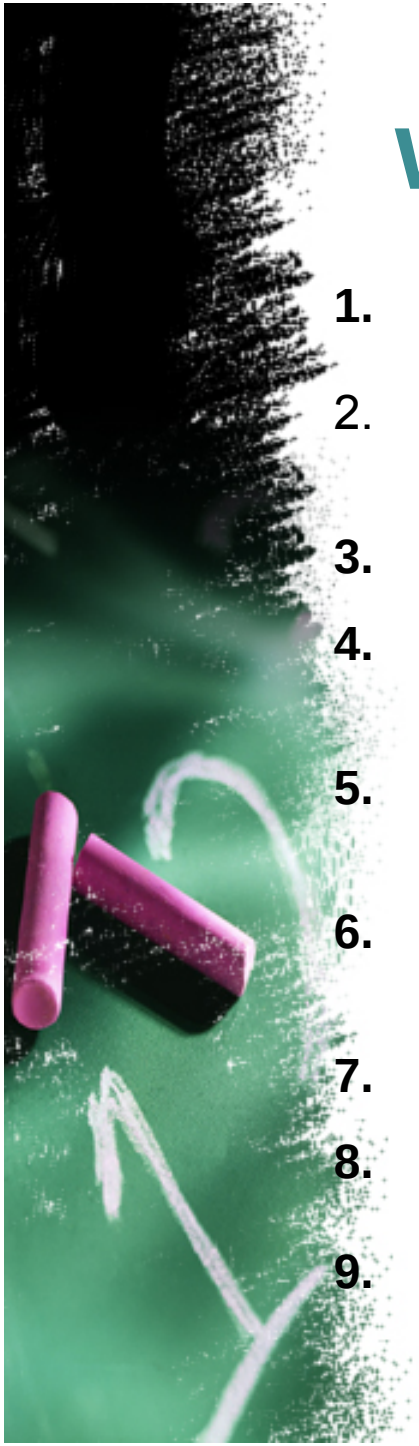
Will be held next year (2015) on

Thursday, November 5th

5th and 6th grade Calcu-Solve Competition

Will be held next year (2015) on

TBD



Warm-Up Answers

1. **6 minutes**; $\frac{1/4}{3/4} = \frac{x}{18}$ so $x = \frac{1}{3} \times 18 = 6$.

2. Ursula ------ Alma ------ Cathy ----- Lani ----- Isabel ----- Betty
   3m *3m* *2m* *2m* *2m*

3. **30;** $3600/2 = 1800,\ \ 1800/3 = 600,\ \ 600/4 = 150,\ \ 150/5 = 30$

4. **11 bicycles and 4 tricycles;** $11 \times 2 + 4 \times 3 = 22 + 12 = 34$ wheels

5. **22 friends and 5 cars;** $3x + 7 = 4x + 2$, so solving for x we get $x = 5$ or 5 cars. Then there are 3 times 5 plus 7 = 22 people.

6. **60;** $15/7$ has a remainder of 1, $30/7$ has a remainder of 2, $45/7$ has a remainder of 3, $60/7$ has a remainder of 4.

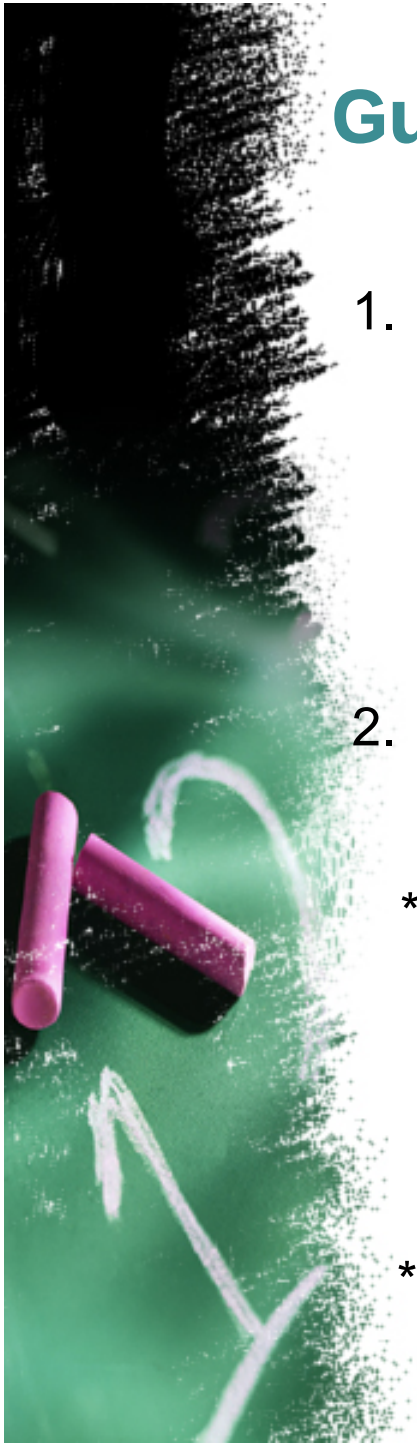
7. **28;** $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$.

8. **400;** $400 + .50 \times 400 + .25 \times 400 = 400 + 200 + 100 = \$700$.

9. **3.9 miles**; $\frac{1.56}{12} = \frac{x}{30}$ and solving gives $x = 30 \times \frac{1.56}{12} = 3.9$.

Schedule and Explanation of Scoring

- **There will be eight Individual Questions.**
- **You will be given 4 minutes to earn 5 points for a correct answer on each Individual Question, or you may wait for a clue, work an extra 3 minutes and earn 3 points for a correct answer on each Individual Question.**
- **There will be two Group Questions. Your team of students will be given 7 minutes to earn 10 points for a correct answer on each Group Question.**
- **After Group Question # 1 and Individual Questions # 1, 2, 3, and 4 we will take a short break.**
- **After the break, we will complete Group Question # 2 and Individual Questions # 5, 6, 7, and 8.**
- **Following Individual Question # 8, we will break for lunch.**
- **If necessary, “tie-breakers” will take place during lunch.**
- **Final scores will be announced and awards will be presented after ties are broken.**
- **Estimated concluding time is 1:30 p.m.**



Guidelines for Tie-Breaking Situations

Individual Tie-Breakers*

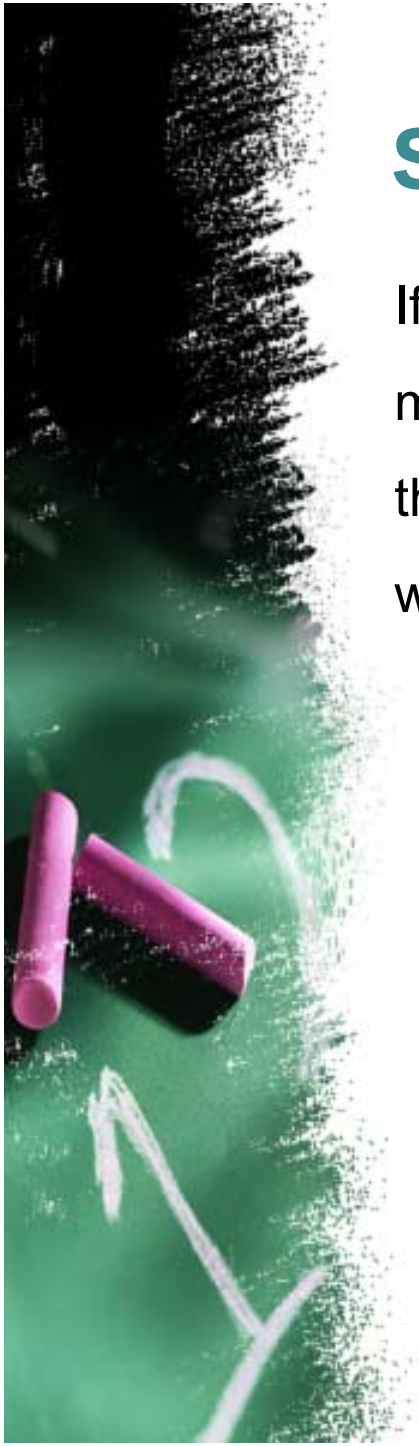
1. In the event of a tied individual score, a sudden death question will be given to those participants who are tied. If an answer is turned in and it is incorrect, the person may continue to work on the problem. The first person with a correct answer within a 5-minute time limit will be declared the winner. If at the end of 5 minutes, no one has submitted a correct answer ...
2. …another sudden death question will be given and step # 1 will be repeated. This procedure will be followed until a winner is determined.

*These rules will be used to determine first, second, third, and tenth place individual winners.

Group Tie-Breakers**

Group tie-breakers will be handled in the same fashion as individual except that the entire group will participate.

**These rules will be used to determine first, second, and third place teams only.



Sample Problem

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Sample Problem - Clue

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Clue: A $6 \times 6 \times 6$ hole is *not* twice as big as a $3 \times 3 \times 3$ hole.

Sample Problem - Solution

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Solution: 16 hours.

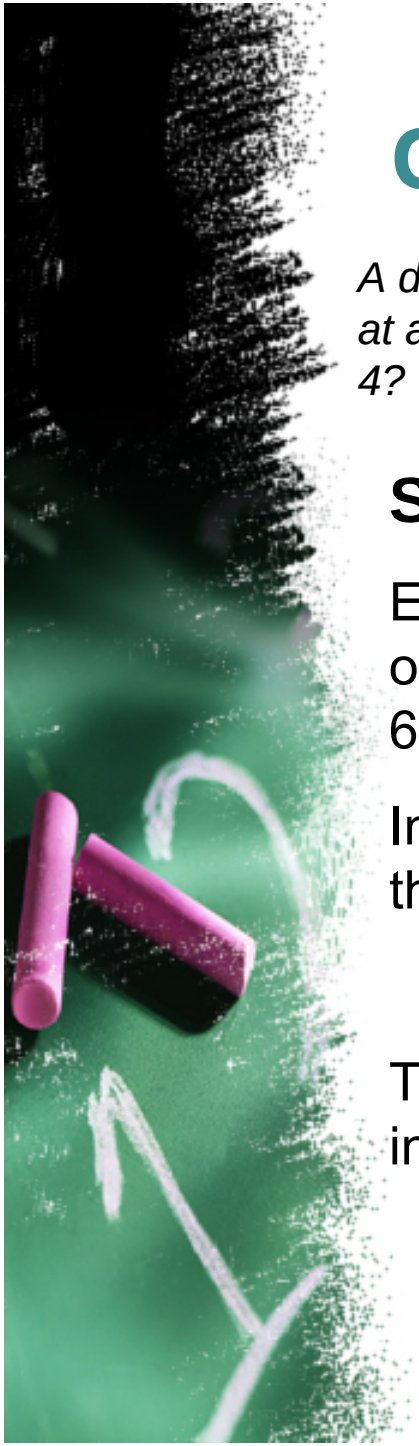
The first hole has volume $3 \times 3 \times 3 = 27$ cubic units. The new hole has volume $6 \times 6 \times 6 = 216$ cubic units. $\frac{216}{27} = 8$, so 8 smaller holes would fit in the new hole, which means it will take $8 \times 2 = 16$ hours to dig the new hole.

Official Competition

Runners: Please pass out Group Question #1 face down and the green Group Answer Sheet #1.

Group Question #1

A digital timer counts down from 6 minutes (6:00) to 0 minutes (0:00) one second at a time. For how many seconds does at least one of the three digits show a 4?



Group Question #1 - Solution

A digital timer counts down from 6 minutes (6:00) to 0 minutes 0:00 one second at a time. For how many seconds does at least one of the three digits show a 4?

Solution:

Each second from time 4:59 to 4:00 has at least one 4 as one of the three digits since the first digit is 4, which gives us 60 seconds.

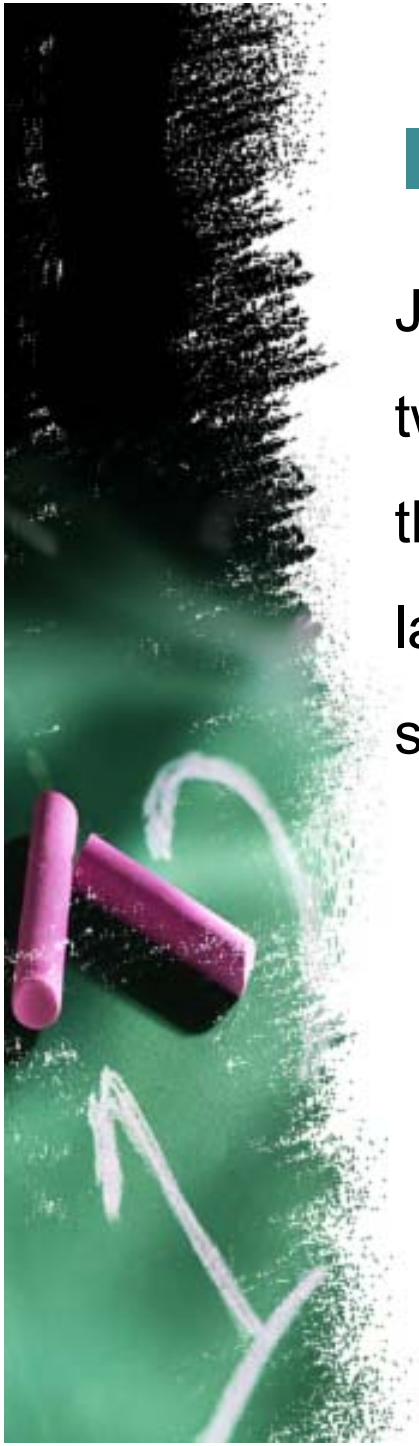
In the first minute, there are 15 seconds when at least one of the digits is a 4:

5:54, 5:49-5:40, 5:34, 5:24, 5:14, and 5:04.

This is the same for the third, fourth, fifth, and sixth minutes in the countdown, so we have a total of

$$60 + 15 \times 5 = 60 + 45 = 135 \text{ seconds}$$

Runners: Please pass out Individual Question #1 face down.



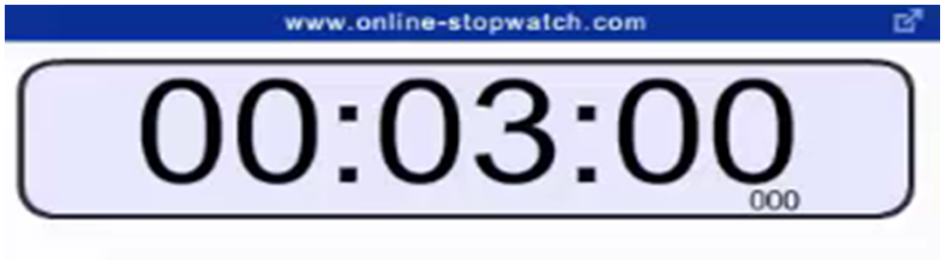
Individual Question #1

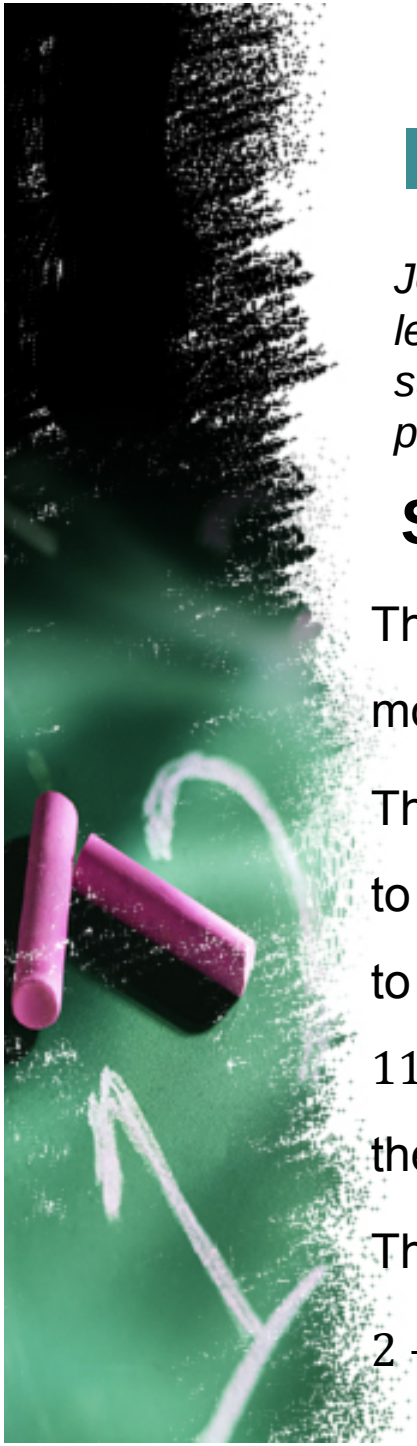
Joy has a cup containing at least one penny, at least two nickels and at least two dimes. The total value of the coins in the cup is $\$1.45$. What is the sum of the largest possible number of nickels in the cup and the smallest possible number of nickels in the cup?

Individual Question #1 - Clue

Joy has a cup containing at least one penny, at least two nickels and at least two dimes. The total value of the coins in the cup is $\$1.45$. What is the sum of the largest possible number of nickels in the cup and the smallest possible number of nickels in the cup?

Clue: Since $\$1.45$ ends in 5, the number of pennies must be a multiple of 5.





Individual Question #1 - Solution

Joy has a cup containing at least one penny, at least two nickels and at least two dimes. The total value of the coins in the cup is $\$1.45$*. What is the sum of the largest possible number of nickels in the cup and the smallest possible number of nickels in the cup?*

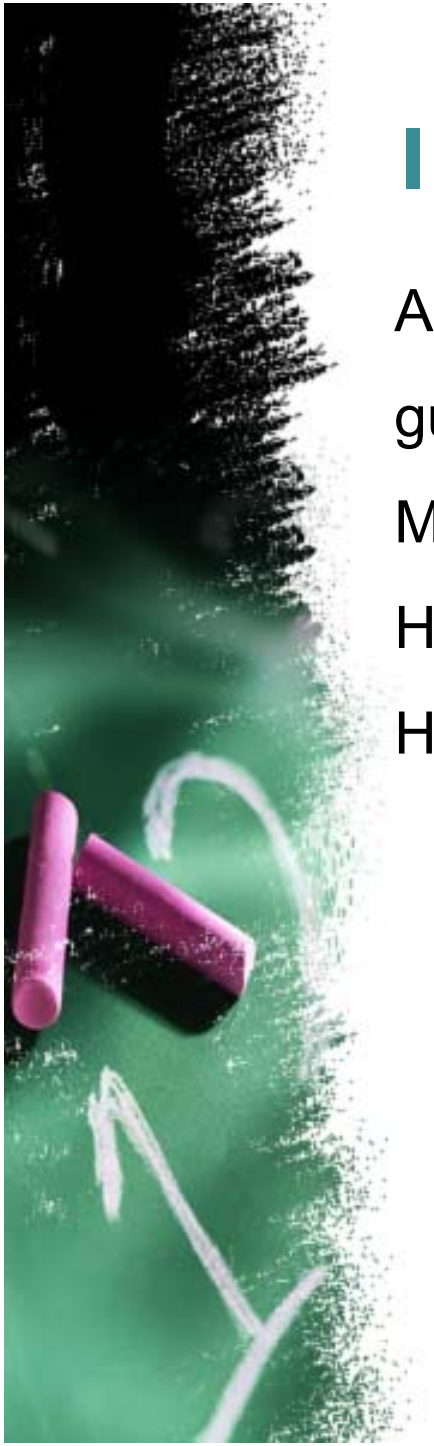
Solution:

The smallest possible number of nickels is 2 because the rest of the money can be made using other coins.

The coins she must have in the cup based on the instructions add up to $1 + 2 \times 5 + 2 \times 10 = 31$ cents. Since $\$1.45$ is a multiple of 5 she has to add 4 pennies to get to 35 cents, then she can get the remaining 110 cents by adding 22 more nickels ($22 \times 5 = 110$ cents). This means the largest possible number of nickels is $22 + 2 = 24$.

The sum of the largest and smallest possible numbers of nickels is $2 + 24 =$ 26 nickels.

Runners: Please pass out Individual Question #2 face down.



Individual Question #2

A pile of gumdrops sits on a table. Bill takes half of the gumdrops plus 7 more. Then Rosie takes 5. Then Michael takes 2 more than a third of what is left. Finally Henrietta takes 6. Four gumdrops remain on the table. How many gumdrops were on the table to start with?

Individual Question #2 - Clue

A pile of gumdrops sits on a table. Bill takes half of the gumdrops plus 7 more. Then Rosie takes 5. Then Michael takes 2 more than a third of what is left. Finally Henrietta takes 6. Four gumdrops remain on the table. How many gumdrops were on the table to start with?

Clue: Work backwards.

Individual Question #2 - Solution

A pile of gumdrops sits on a table. Bill takes half of the gumdrops plus 7 more. Then Rosie takes 5. Then Michael takes 2 more than a third of what is left. Finally Henrietta takes 6. Four gumdrops remain on the table. How many gumdrops were on the table to start with?

Solution:

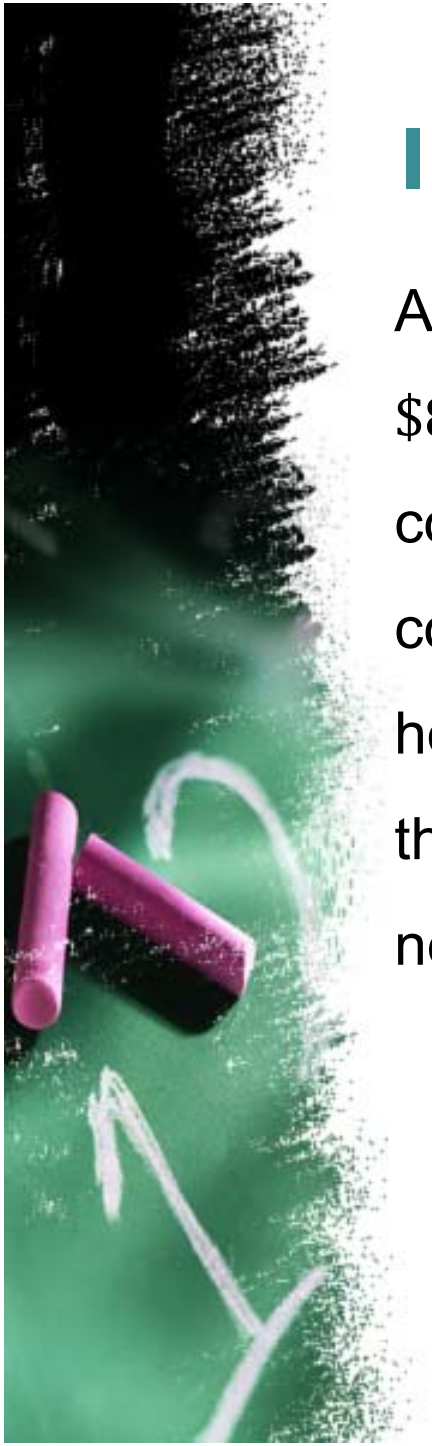
Since there are 4 gumdrops remaining on the table after Henrietta takes 6, there must have been $4 + 6 = 10$ before she took hers.

If we add back the extra 2 gumdrops Michael took, we get $10 + 2 = 12$, which is $\frac{2}{3}$ of what was on the table before he took any. If 12 is two thirds, then 6 is one third, so there were $12 + 6 = 18$.

Before Rosie took her 5 there were $18 + 5 = 23$ gumdrops.

If we add back the extra 7 gumdrops Bill took, we get $23 + 7 = 30$, which is $\frac{1}{2}$ of what was on the table to start with. So there were $30 + 30 = 60$ gumdrops on the table.

Runners: Please pass out Individual Question #3 face down.



Individual Question #3

A store purchases game consoles from a factory for $\$85.89$ each. The store normally sells one of these consoles for 230% of the factory cost, but a store coupon gives 25% off this selling price. Ignoring tax, how much does a customer with this coupon pay for the game console? Express your answer to the nearest hundredth.

Individual Question #3 - Clue

A store purchases game consoles from a factory for 85.89 each. The store normally sells one of these consoles for 230% of the factory cost, but a store coupon gives 25% off this selling price. Ignoring tax, how much does a customer with this coupon pay for the game console? Express your answer to the nearest hundredth.

Clue: 230% is 2.3 as a decimal.

Individual Question #3 - Solution

A store purchases game consoles from a factory for $85.89 each. The store normally sells one of these consoles for 230% of the factory cost, but a store coupon gives 25% off this selling price. Ignoring tax, how much does a customer with this coupon pay for the game console? Express your answer to the nearest hundredth.

Solution:

The store sells the game consoles for 2.3 times the price they paid the factory, which is

$$2.3 \times (\$85.89) = \$197.55$$

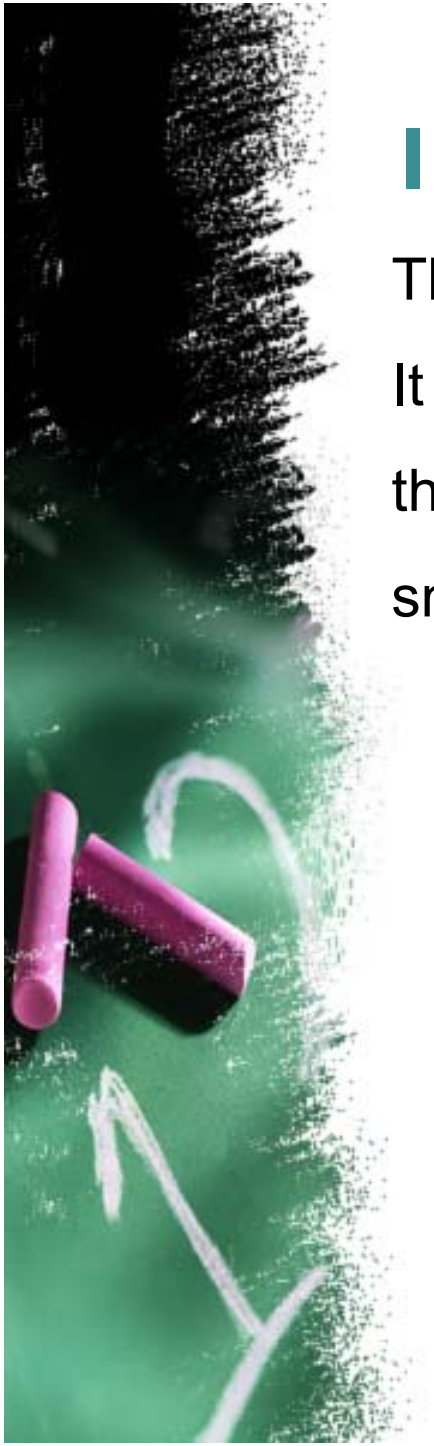
The customer's coupon saves them 25% of this price or

$$0.25 \times (\$197.55) = \$49.39$$

So the final price the customer pays is

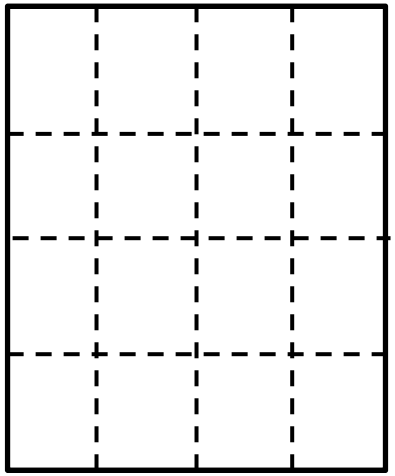
$$\$197.55 - \$49.39 = \$148.16$$

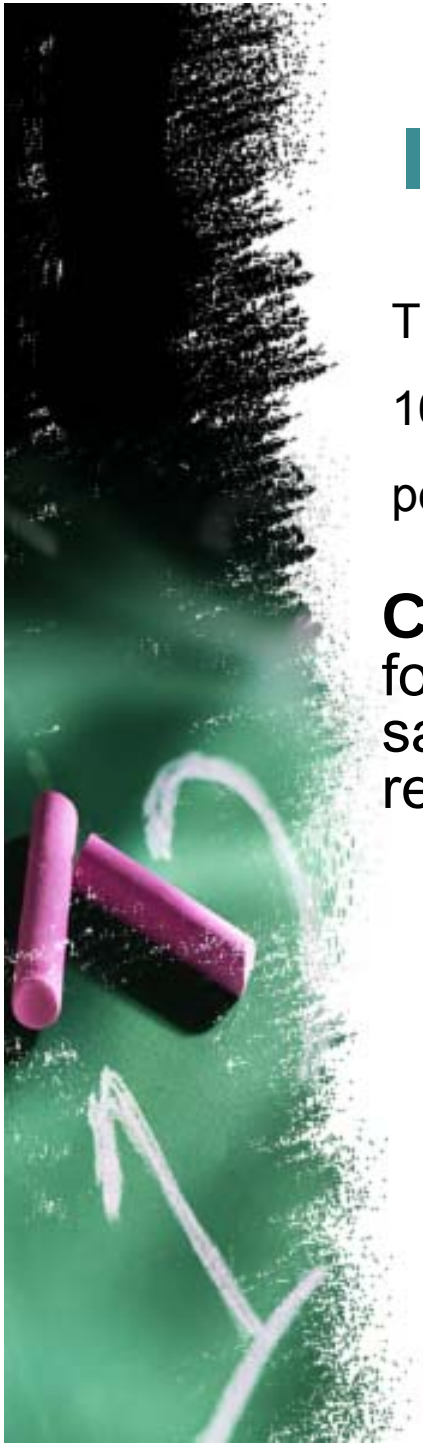
Runners: Please pass out Individual Question #4 face down.



Individual Question #4

The perimeter of a rectangular piece of paper is 90 cm. It is cut into 16 congruent rectangles as shown. What is the total of the sum of the perimeters of the sixteen smaller rectangles?

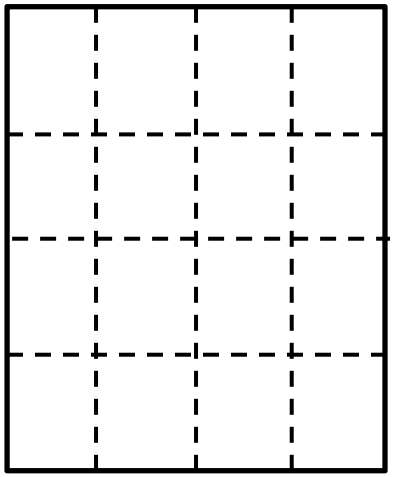


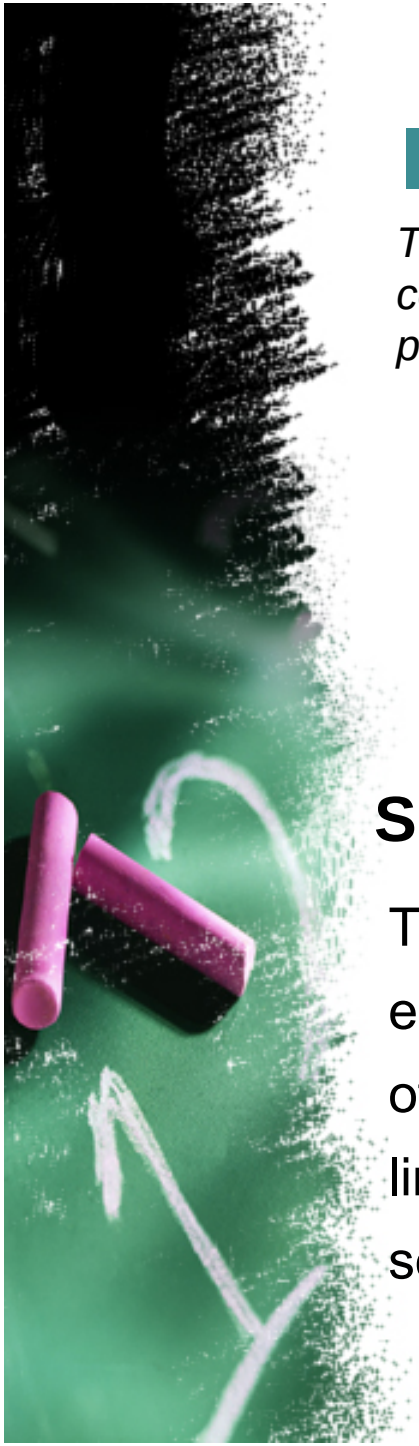


Individual Question #4 - Clue

The perimeter of a rectangular piece of paper is 90 cm. It is cut into 16 congruent rectangles as shown. What is the total of the sum of the perimeters of the sixteen smaller rectangles?

Clue: The sum of the lengths of four of the small rectangles is the same as the length of the original rectangle.





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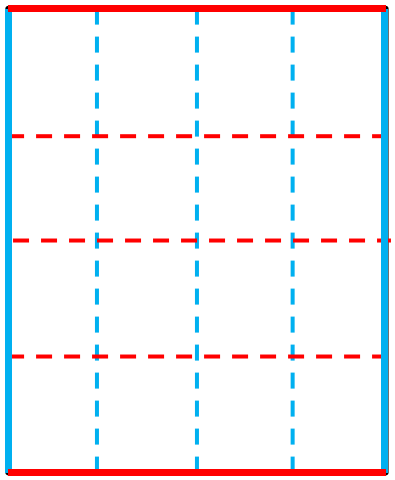


Solution:

The lengths of two of the blue lines plus two of the red lines equals the perimeter of the original rectangle. The total sum of the perimeters is the sum of the lengths of the five blue lines and the five red lines. Five is 2.5 times as large as two, so the total sum of the perimeters is

$$2.5 \times (90\ cm) = 225\ cm.$$

Snack Time

Runners: Please pass out Group Question #2 face down and the green Group Answer Sheet #2.

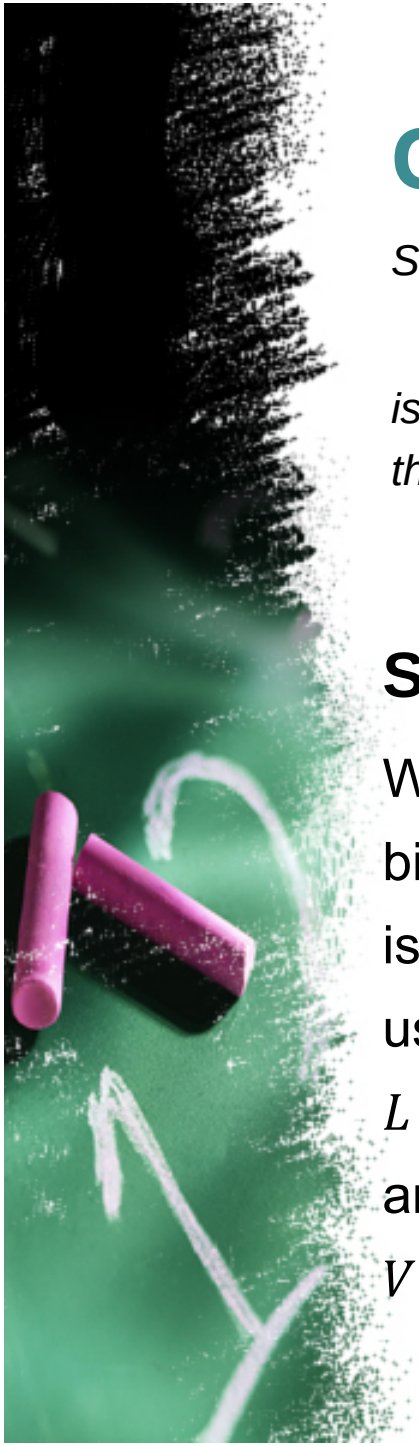
Group Question #2

Suppose that each distinct letter in the equation

$$CAL + CUS = OLVE$$

is replaced by a different digit chosen from 1 through 9 in such a way that the resulting equation is true. If $E = 6$, what is the value of $L + V$?

$$\begin{array}{cccc} & C & A & L \\ + & C & U & S \\ \hline O & L & V & E \end{array}$$



Group Question #2 - Solution

Suppose that each distinct letter in the equation

$$CAL + CUS = OLVE$$

is replaced by a different digit chosen from 1 through 9 in such a way that the resulting equation is true. If $E = 6$, what is the value of $L + V$?

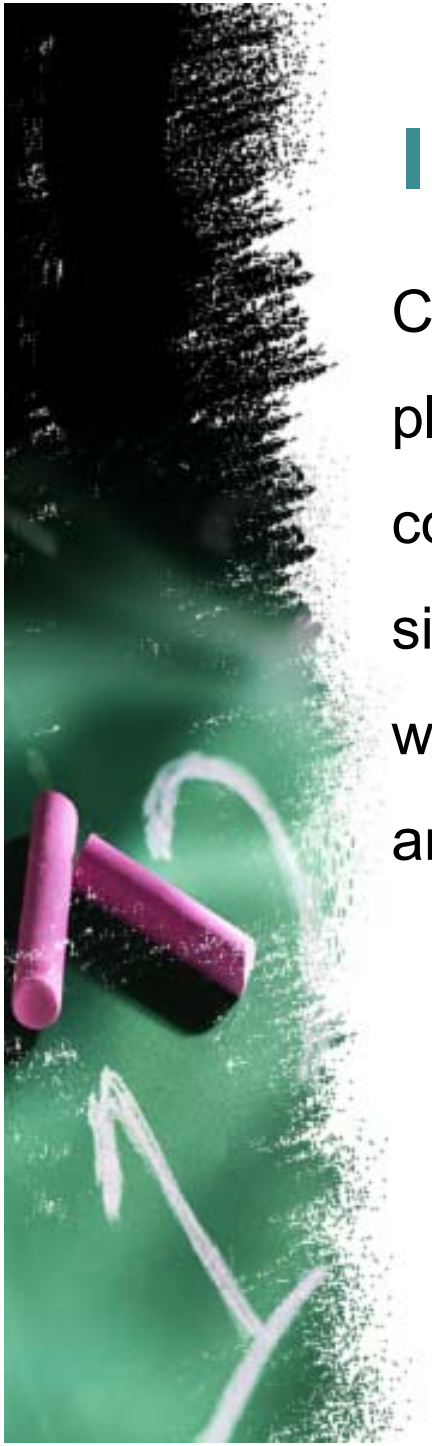
$$\begin{array}{cccc} & C & A & L \\ + & C & U & S \\ \hline O & L & V & E \end{array}$$

Solution:

We are told that $E = 6$. O must equal 1, so C must be 5 or bigger. We can't have $C = 5$ otherwise L would be 0, which isn't allowed, and we can't have $C = 6$ because 6 is already used. So C is 7, 8, or 9. Let's try $C = 7$. That would make $L = 4$, so S would have to be 2. Now the only digits remaining are 3, 5, 8, and 9. $3 + 5 = 8$, so if we make $A = 3$, $U = 5$, and $V = 8$ then the equation will be true. So

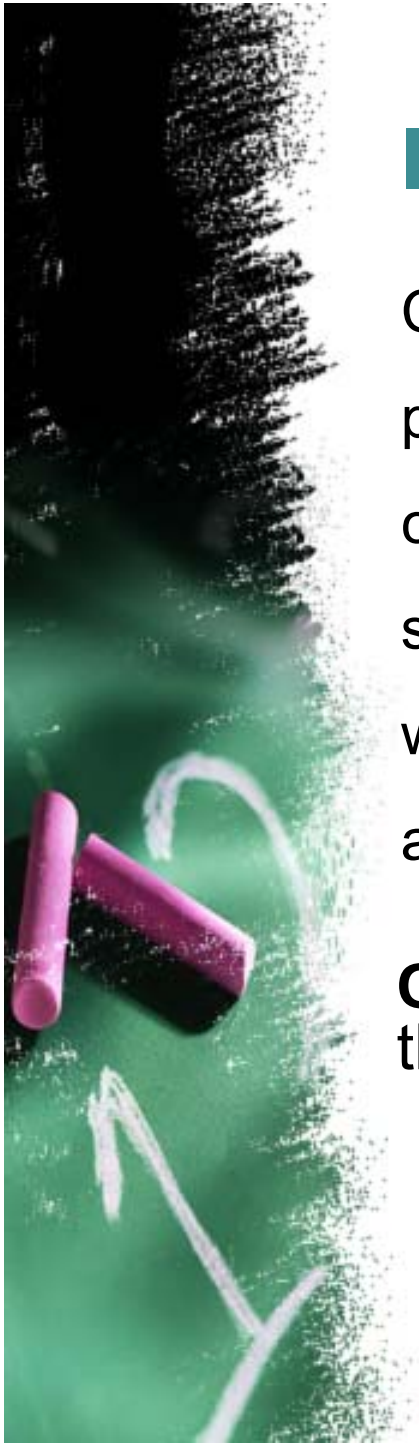
$$L + V = 4 + 8 = \mathbf{12}$$

Runners: Please pass out Individual Question #5 face down.



Individual Question #5

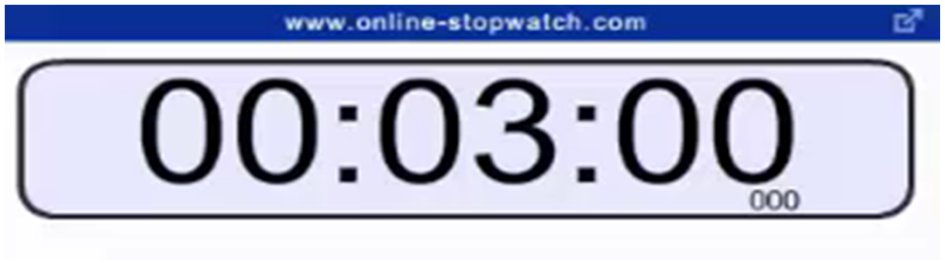
Cool Designs Sign company charges an $85 set-up fee plus $7 for each sign printed. Signs And More Sign company has no set-up fee, but charges $11 for each sign printed. What is the maximum number of signs for which a customer would save money by using Signs and More?

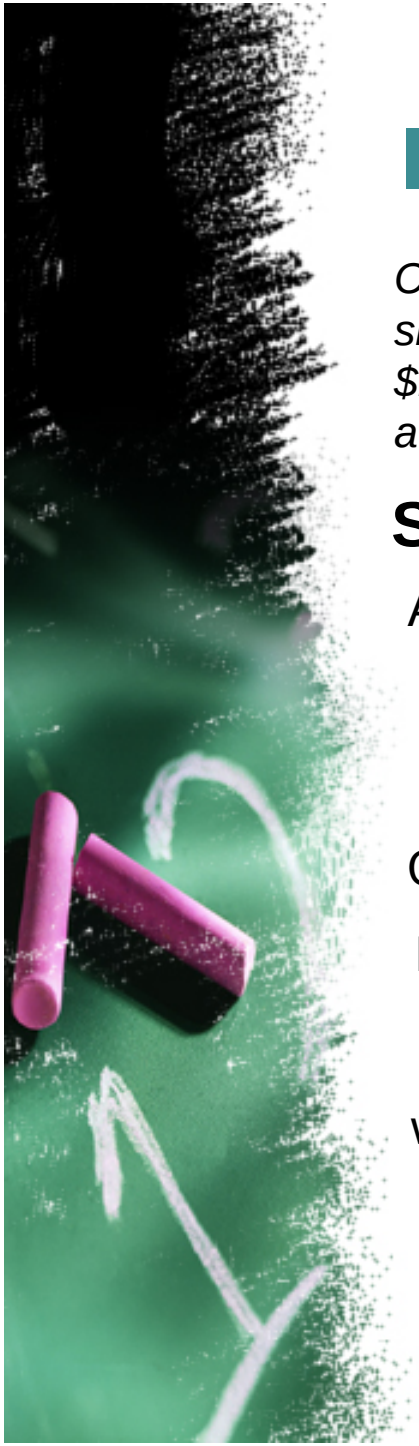


Individual Question #5 - Clue

Cool Designs sign company charges an $85 set-up fee plus $7 for each sign printed. Signs And More sign company has no set-up fee, but charges $11 for each sign printed. What is the maximum number of signs for which a customer would save money by using Signs and More?

Clue: How much more do they spend per sign if they buy from Signs And More?





Individual Question #5 - Solution

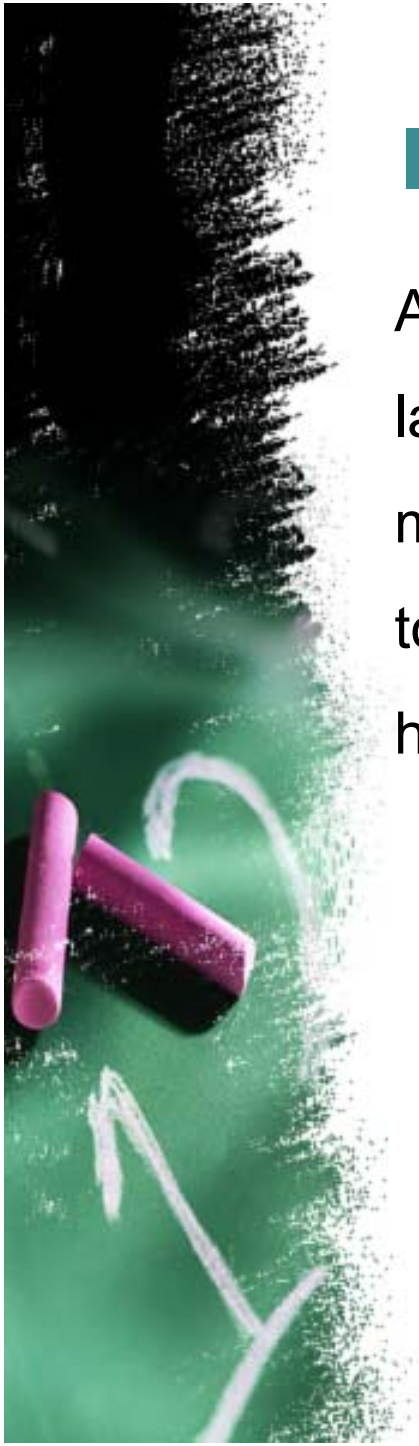
Cool Designs sign company charges an $85 set-up fee plus $7 for each sign printed. Signs And More sign company has no set-up fee, but charges $11 for each sign printed. What is the maximum number of signs for which a customer would save money by using Signs and More?

Solution:

A customer will spend an extra $\$11 - \$7 = \$4$ per sign if they buy from Signs and More. If they buy 21 signs from Signs and More they will spend an extra $21 \times \$4 = \$84$, which is still less than Cool Design's set-up fee. But if they buy 22 signs from Signs and More they will spend an extra $22 \times \$4 = \$88$, which is more than Cool Design's set-up fee. So the maximum number of signs for which a customer would save money by using Signs and More is

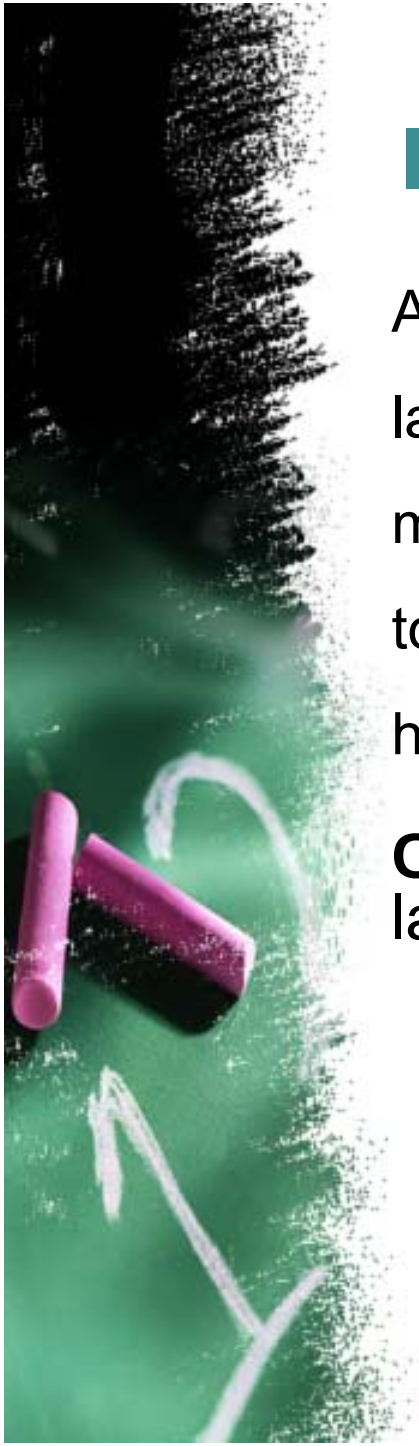
21 signs.

Runners: Please pass out Individual Question #6 face down.



Individual Question #6

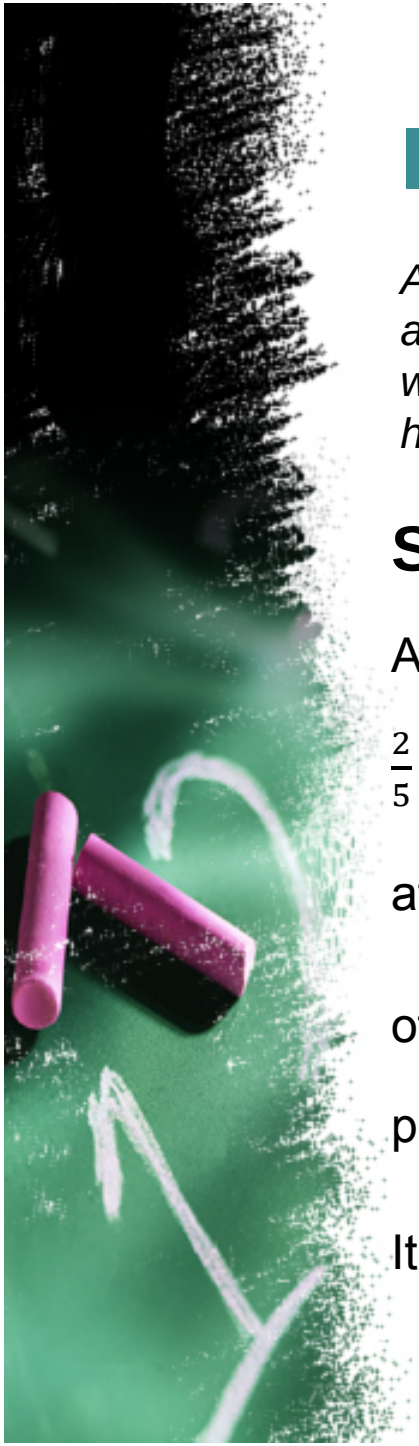
Andrew can mow a lawn in 1 hour, Samuel can mow two lawns in 5 hours, and TJ can mow ½ a lawn in 45 minutes. How long will it take all three working together to mow 3 lawns? Express your answer to the nearest hundredth.



Individual Question #6 - Clue

Andrew can mow a lawn in 1 hour, Samuel can mow two lawns in 5 hours, and TJ can mow ½ a lawn in 45 minutes. How long will it take all three working together to mow 3 lawns? Express your answer to the nearest hundredth.

Clue: Express each person's rate in terms of lawns per hour.



Individual Question #6 - Solution

Andrew can mow a lawn in 1 hour, Samuel can mow two lawns in 5 hours, and TJ can mow ½ a lawn in 45 minutes. How long will it take all three working together to mow 3 lawns? Express your answer to the nearest hundredth.

Solution:

Andrew mows at a rate of 1 lawn per hour. Samuel mows at a rate of $\frac{2}{5}$ lawn per hour. TJ can mow $\frac{1}{2}$ of a lawn in $\frac{3}{4}$ of an hour, so he mows at a rate of $\frac{1/2}{3/4} = \frac{2}{3}$ lawn per hour. So all together they mow at a rate of $1 + \frac{2}{5} + \frac{2}{3} = \frac{31}{15}$ lawns per hour, which means it takes them $\frac{15}{31}$ hour per lawn.

It will take them $\frac{15}{31} \times 3 = \frac{45}{31} = 1.45$ hours to mow 3 lawns.

Runners: Please pass out Individual Question #7 face down.

Individual Question #7

Natalie gives clues about her six-digit secret number:

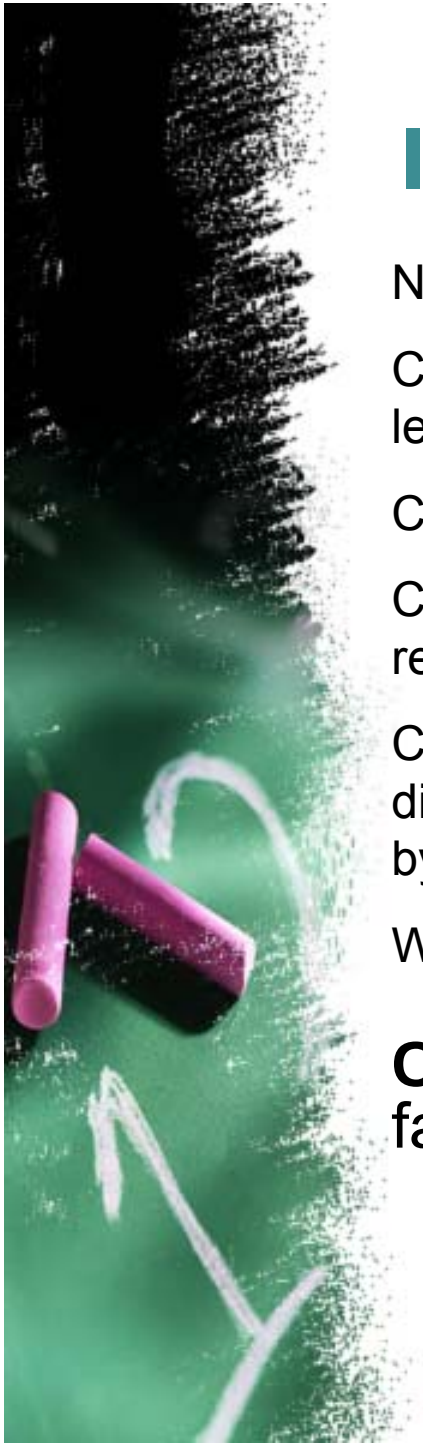
Clue 1: It is the same number if you read it from right to left as from left to right.

Clue 2: The number is a multiple of 5.

Clue 3: Cross off the first two digits and the last two digits. The remaining two digit number has only one prime factor.

Clue 4: The first two digits (reading from left to right) form a two digit prime number that is larger than the two digit number formed by the last two digits.

What is Natalie's six-digit number?



Individual Question #7 - Clue

Natalie gives clues about her six-digit secret number:

Clue 1: It is the same number if you read it from right to left as from left to right.

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Clue 4: The first two digits (reading from left to right) form a two digit prime number that is larger than the two digit number formed by the last two digits.

What is Natalie's six-digit number?

Clue: A two digit number with only one prime factor must be a prime number.

Individual Question #7 - Solution

Natalie gives clues about her six-digit secret number:

Clue 1: It is the same number if you read it from right to left as from left to right.

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What is Natalie's six-digit number?

Solution:

Clue 1 tells us that it looks like □ □ □ □ □ □ .

Clue 2 tell us that the last digit must be a 0 or a 5. Since the first digit must be the same and we want a 6 digit number we know it must be 5, so now the number looks like 5 □ □ □ □ 5.

Clue 3 tells us that □ □ has only one prime factor and 11 is the only two digit prime number that is prime, so now the number looks like

5 □ 1 1 □ 5.

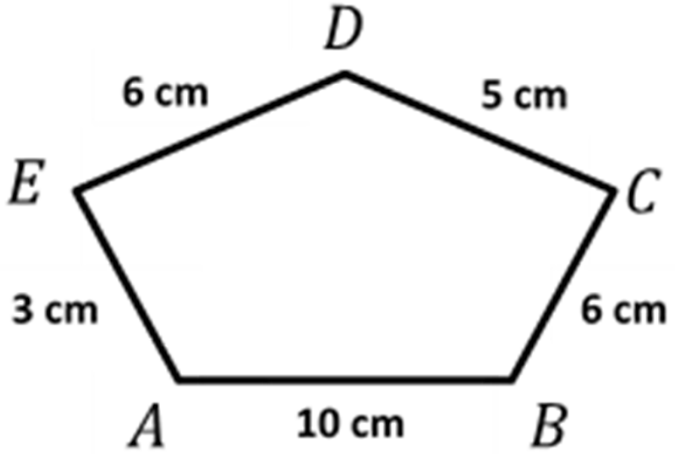
Clue 4 tells us that 5 □ is bigger than □ 5. That means □ could be 0, 1, 2, 3, or 4. Clue 4 also tells us that 5 □ is prime, so □ must be 3.

Natalie's number is 5 3 1 1 3 5.

Runners: Please pass out Individual Question #8 face down.

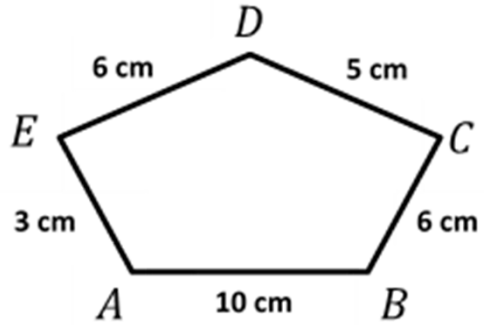
Individual Question #8

Two ladybugs walk from point A to point E along the sides of figure ABCDE. They start and finish together. The first ladybug walks from A to B to C to D to E at an average speed of 3 centimeters per second. The second ladybug walks directly from A to E. What is the average speed of the second ladybug?



Individual Question #8 - Clue

Two ladybugs walk from point A to point E along the sides of figure ABCDE. They start and finish together. The first ladybug walks from A to B to C to D to E at an average speed of 3 centimeters per second. The second ladybug walks directly from A to E. What is the average speed of the second ladybug?

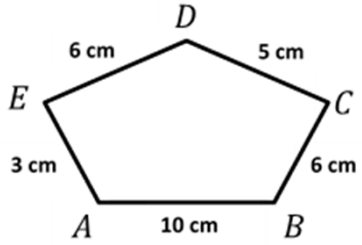


Clue:

The second ladybug is much slower than the first one.

Individual Question #8 - Solution

Two ladybugs walk from point A to point E along the sides of figure ABCDE. They start and finish together. The first ladybug walks from A to B to C to D to E at an average speed of 3 centimeters per second. The second ladybug walks directly from A to E. What is the average speed of the second ladybug?



Solution:

The first ladybug walks $10\ cm + 6\ cm + 5\ cm + 6\ cm = 27\ cm$ at $3\ cm$ per second, so it takes the first ladybug $\frac{27}{3} = 9$ sec to walk the entire way from A to E. The second ladybug takes the same amount of time so it walks $3\ cm$ in $9\ \text{sec}$ and its average speed is

$$\frac{3cm}{9sec} = \frac{1}{3} \text{ cm per second.}$$

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