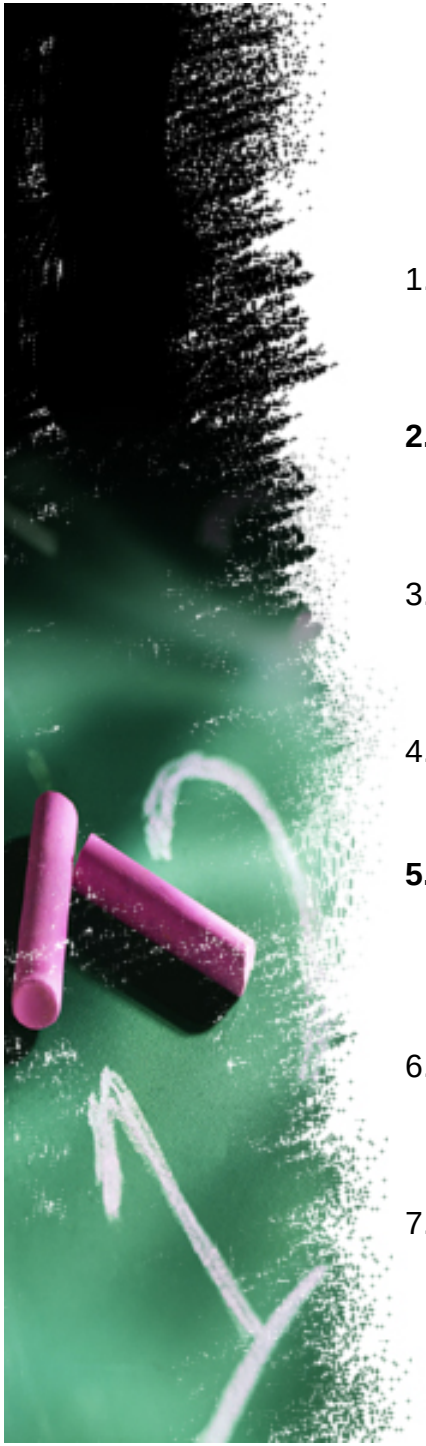
Welcome to the 2014 Calcu-Solve 7th / 8th Grade Competition!

Northwest Pennsylvania Division



We hope you have a challenging and successful day!
While we are waiting for all the teams to arrive, please:

1. Put your coats and lunches in an area where your team sponsors can keep an eye on them. The only things you need to take to your team's table are your calculators, and sharpened pencils (if you brought any.) Sit at the table with your team number on it.

2. **Make sure your team sponsor has completed the registration/scoring card that is in the folder he/she received when you arrived. This card should be filled out completely and given to the Registration Desk.**

3. Make a nametag for everyone in your group. Include your first and last names, school district, school name, and team number on the tag. Please wear the nametag during the entire competition.

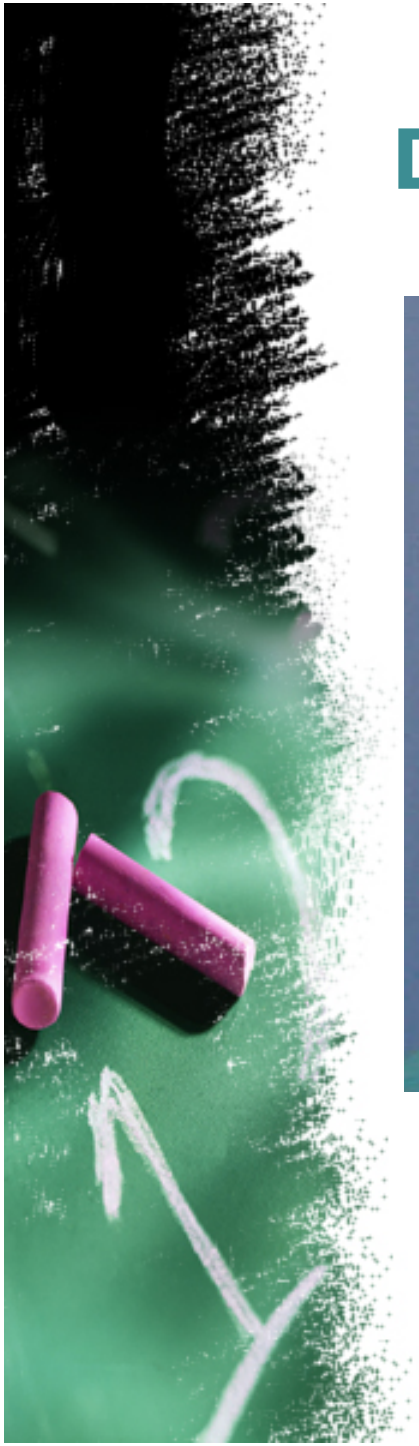
4. Read over the information in the folder with your team sponsor. The rules and scoring procedures are explained. We will review these briefly just before the competition begins.

5. **Each person on your team should take one stapled packet of individual answer sheets from the team folder and fill in your full name and team number on EVERY sheet. Print neatly and accurately! Your team number is displayed on the sign at your table and on your team folder.**

6. Begin to practice for the competition by working on the Warm-Up Questions that are in your folder. We will go over the answers to these questions just before we begin the actual competition questions.

7. If you need help or further direction, please find a Student Assistant in a green shirt or see Dr. Mr. Bancroft or Dr. Mrs. Bancroft.

Relax, Have Fun, and Good Luck!

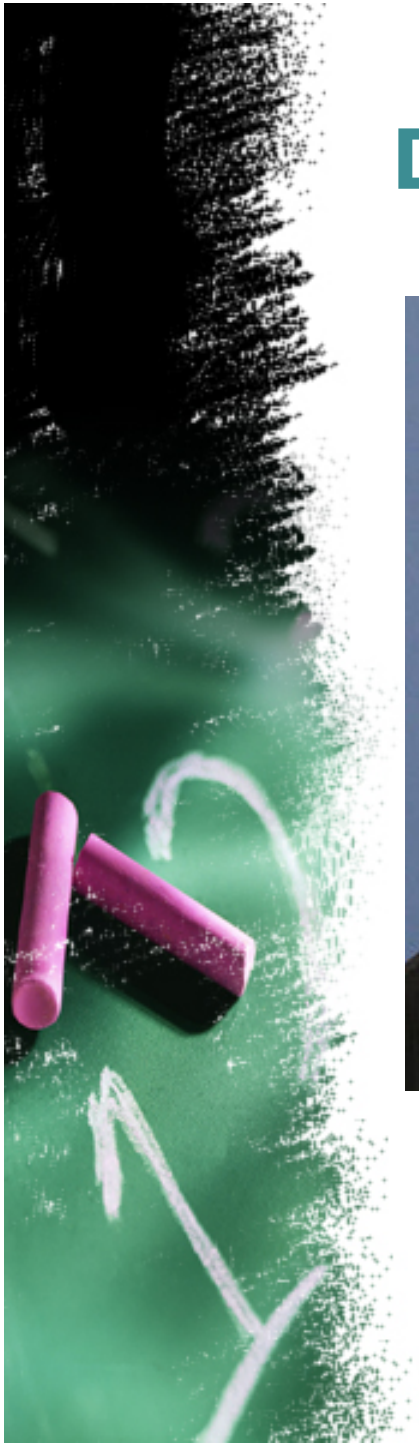


Dr. Erin Bancroft

- Program Director
- Assistant Professor at GCC

Mrs. Sarah Potter

- Assistant Director
- Adjunct Professor at GCC



Dr. Eric Bancroft

- Assistant Director
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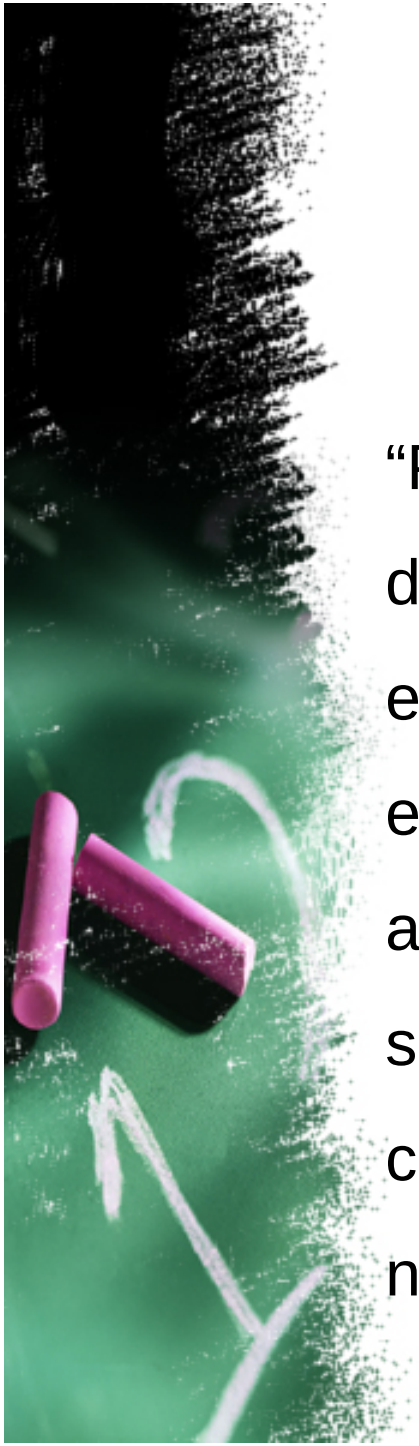


II-VI Foundation

“Programs supported by the II-VI Foundation are designed to create a stronger population of new engineering-, science-, and mathematics-educated individuals that will ultimately increase and improve the pool of engineers and scientists seeking to tackle the tough and ever more complicated technical problems facing our nation and the world.”

“Thank you” to…

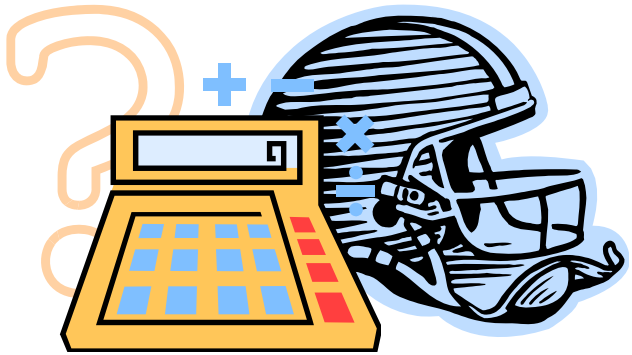
- II-VI Foundation
- Roxann Williams

…and all of our GCC student volunteers!

- Anna O’Neil
- Julia Berek
- Shelby Davenport
- Sarah Deemer
- Katy Gibson
- Kendra Helfrich
- Annie Laurie Holfelder
- Amanda Johnson
- Rachel Lapp
- Robin Lawson
- Hannah Liermann
- Kelly Scobee
- David Shang
- Abby Slater
- Bonnie Stahl
- Rebekah Thomas
- Kailey Tuhacek
- Brittany Turner
- Hannah Wierenga
- Cindy Zohoranacky

Calcu-Solve Super Bowl!

Winning teams and individuals from our competition will be invited to compete against the winners from two other regional competitions at Duquesne University in February.



7th and 8th grade Calcu-Solve Competition

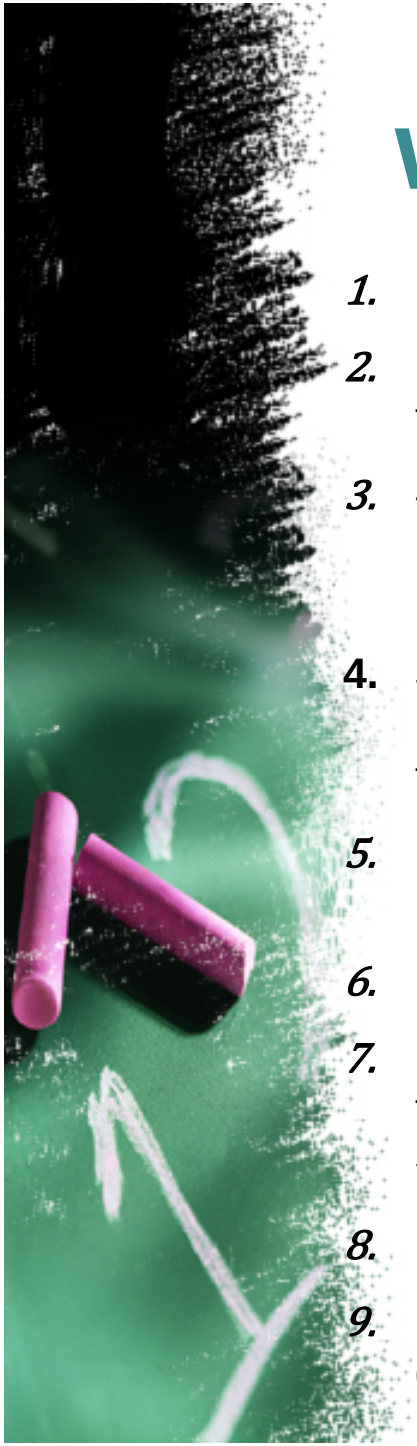
Will be held next year (2015) on

Thursday, November 5th

5th and 6th grade Calcu-Solve Competition

Will be held next year (2015) on

TBD

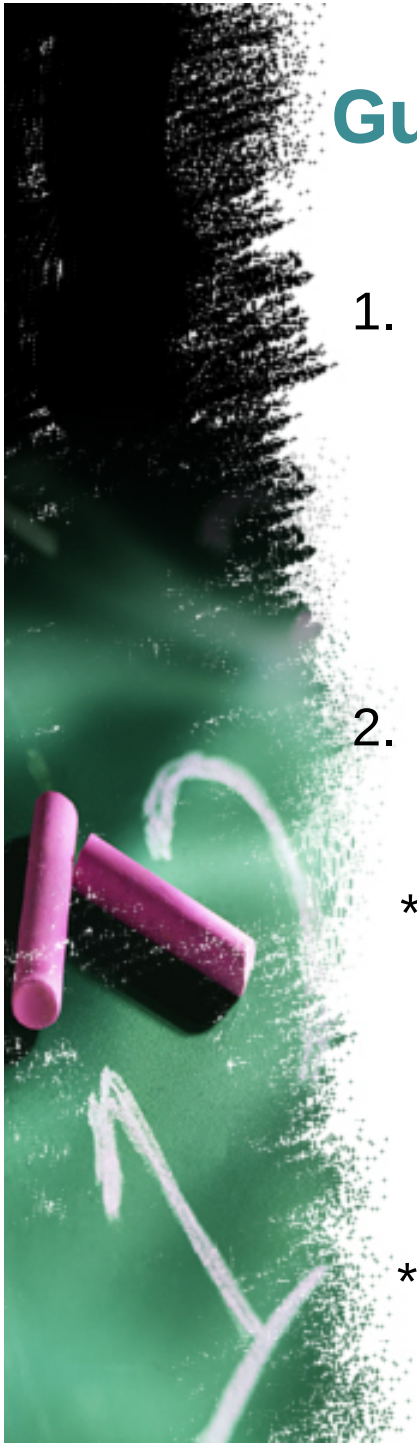


Warm-Up Answers

1. **$\$1.05$;** $x + (1.00 + x) = 1.10$ so $x = 0.05$ and pen costs $\$1.05$.
2. **10;** The median is the middle number so $n + 6 = 9$ and $n = 3$. Adding the terms and dividing by 5 we find that the mean is 10.
3. **4**; The first eight numbers are $1 - 8$ and the next 8 numbers are $9 - 2$ and this pattern of the same 16 number continues. Dividing 100 by 16 gives us 6 with a remainder of 4 so the next number is the fourth in the pattern which is 4.
4. **Sunday**; $1 + 2 + 4 + 8 + 16 + 32 + 64 = 127$ and $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 = 255$ so seven days after she started is the first time she has more than $\$2$.
5. **$\$28$;** A ration of 5:4:2 means that there are 11 parts in the whole and Michael received $4/11$ of $\$77$ which is $\$28$.
6. **30;** The difference between $14^2 = 196$ and $16^2 = 256$ is 60 and $14 + 16 = 30$.
7. **25 miles;** Andrea's house, Sunville and Happytown form a right triangle with the Sunville and Happytown as the vertices of the hypotenuse. Thus $d = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$.
8. **3;** Since $3^4 = (3^2)^2 = 9^2$ we have $9^2 = 9^{5-d}$ and $2 = 5 - d$. Thus $d = 3$.
9. **$\$155.40$;** The first discount makes the bike $75\% \times 259 = \$194.25$. The second discount makes the bike $80\% \times 194.25 = \$155.40$.

Schedule and Explanation of Scoring

- **There will be eight Individual Questions.**
- **You will be given 4 minutes to earn 5 points for a correct answer on each Individual Question, or you may wait for a clue, work an extra 3 minutes and earn 3 points for a correct answer on each Individual Question.**
- **There will be two Group Questions . Your team of students will be given 7 minutes to earn 10 points for a correct answer on each Group Question.**
- **After Group Question # 1 and Individual Questions # 1, 2, 3, and 4 we will take a short break.**
- **After the break, we will complete Group Question # 2 and Individual Questions # 5, 6, 7, and 8.**
- **Following Individual Question # 8, we will break for lunch.**
- **If necessary, “tie-breakers” will take place during lunch.**
- **Final scores will be announced and awards will be presented after ties are broken.**
- **Estimated concluding time is 1:30 p.m.**



Guidelines for Tie-Breaking Situations

Individual Tie-Breakers*

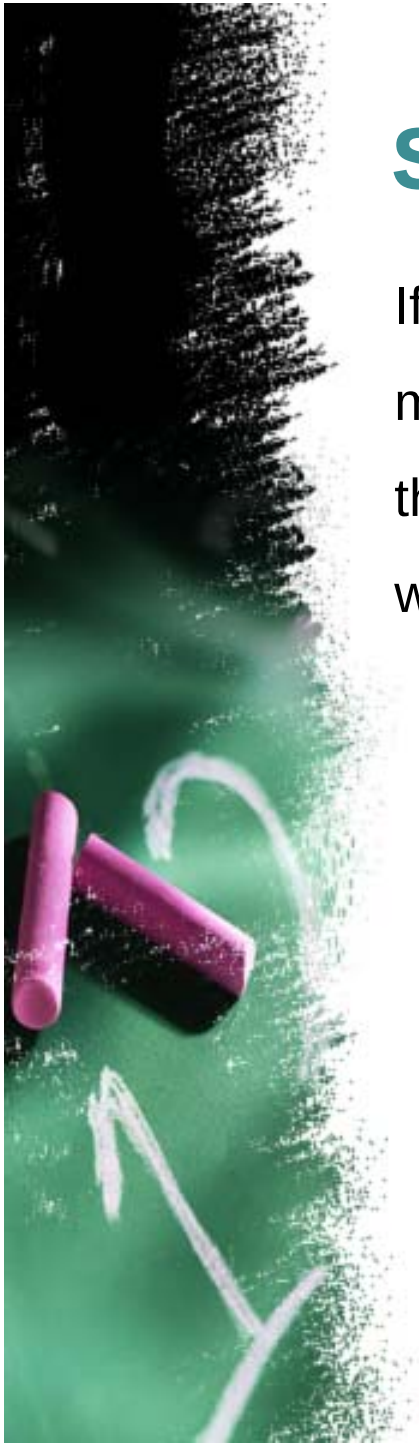
1. In the event of a tied individual score, a sudden death question will be given to those participants who are tied. If an answer is turned in and it is incorrect, the person may continue to work on the problem. The first person with a correct answer within a 5-minute time limit will be declared the winner. If at the end of 5 minutes, no one has submitted a correct answer ...
2. …another sudden death question will be given and step # 1 will be repeated. This procedure will be followed until a winner is determined.

*These rules will be used to determine first, second, third, and tenth place individual winners.

Group Tie-Breakers**

Group tie-breakers will be handled in the same fashion as individual except that the entire group will participate.

**These rules will be used to determine first, second, and third place teams only.



Sample Problem

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Sample Problem - Clue

If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Clue: A $6 \times 6 \times 6$ hole is *not* twice as big as a $3 \times 3 \times 3$ hole.

Sample Problem - Solution

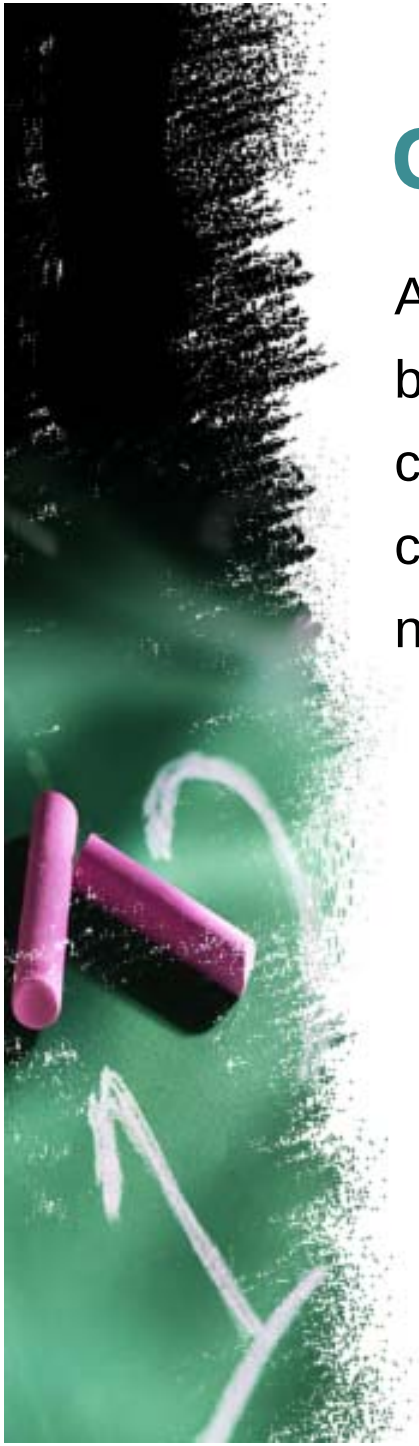
If it takes two men two hours to dig a hole 3 meters long, 3 meters wide, and 3 meters deep, then how long will it take the same two men to dig a hole 6 meters long, 6 meters wide, and 6 meters deep if they work at the same rate?

Solution: 16 hours.

The first hole has volume $3 \times 3 \times 3 = 27$ cubic units. The new hole has volume $6 \times 6 \times 6 = 216$ cubic units. $\frac{216}{27} = 8$, so 8 smaller holes would fit in the new hole, which means it will take $8 \times 2 = 16$ hours to dig the new hole.

Official Competition

Runners: Please pass out Group Question #1 face down and the green Group Answer Sheet #1.



Group Question #1

A standard deck of playing cards with 26 red cards and 26 black cards is split into two piles, each having at least one card. In pile A there are six times as many black cards as red cards. In pile B, the number of red cards is a multiple of the number of black cards. How many red cards are in pile B?

Group Question #1 - Solution

A standard deck of playing cards with 26 red cards and 26 black cards is split into two piles, each having at least one card. In pile A there are six times as many black cards as red cards. In pile B, the number of red cards is a multiple of the number of black cards. How many red cards are in pile B?

Solution:

	Pile A	Pile B
Red	x	$26 - x$
Black	$6x$	$26 - 6x$

Since there are only 26 total black cards we know that $1 \leq x \leq 4$. For each of these values of x we can check to see if the number of red cards in Pile B is a multiple of the number of black cards in Pile B.

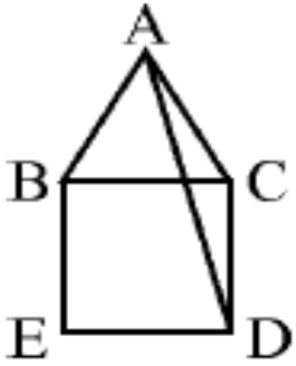
x	Red	Black
1	$26 - 1 = 25$	$26 - 6(1) = 20$
2	$26 - 2 = 24$	$26 - 6(2) = 14$
3	$26 - 3 = 23$	$26 - 6(3) = 8$
4	$26 - 4 = 22$	$26 - 6(4) = 2$

Since 22 is a multiple of 2, Pile B has **22 red cards**.

Runners: Please pass out Individual Question #1 face down.

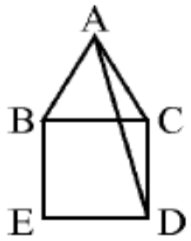
Individual Question #1

Equilateral triangle ABC and square BCDE share the edge BC, as shown. What is the number of degrees in the measure of angle CAD?

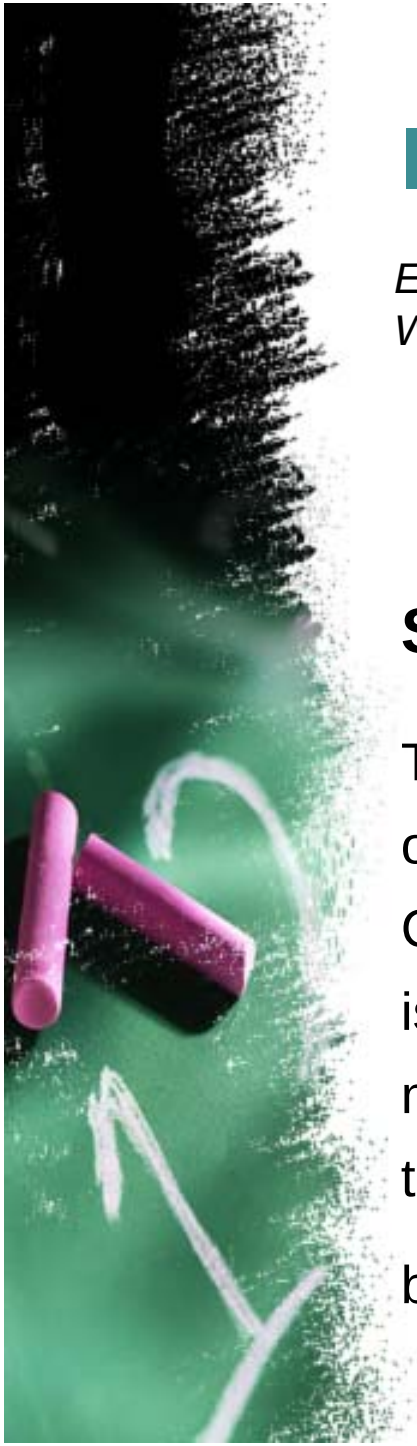


Individual Question #1 - Clue

Equilateral triangle ABC and square BCDE share the edge BC, as shown. What is the number of degrees in the measure of angle CAD?

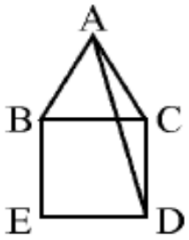


Clue: All of the interior angles in an equilateral triangle are 60°.



Individual Question #1 - Solution

Equilateral triangle ABC and square BCDE share the edge BC, as shown. What is the number of degrees in the measure of angle CAD?



Solution:

The fact that ABC and BCDE share an edge tells us that all of their sides are the same, which means edge AC and edge CD have the same length. Thus, triangle ACD is an isosceles triangle and angles CAD and CDA have the same measure. Since angle ACD is $60 + 90 = 150°$ and we know that the angles in a triangle add up to $180°$, angle CAD must be $\frac{180° - 150°}{2} = \mathbf{15°}$.

Runners: Please pass out Individual Question #2 face down.

Individual Question #2

What is the units digit when 7^{2014} is multiplied out?

Individual Question #2 - Clue

What is the units digit when 7^{2014} is multiplied out?

Clue: Look for a pattern.

Individual Question #2 - Solution

What is the units digit when 7^{2014} is multiplied out?

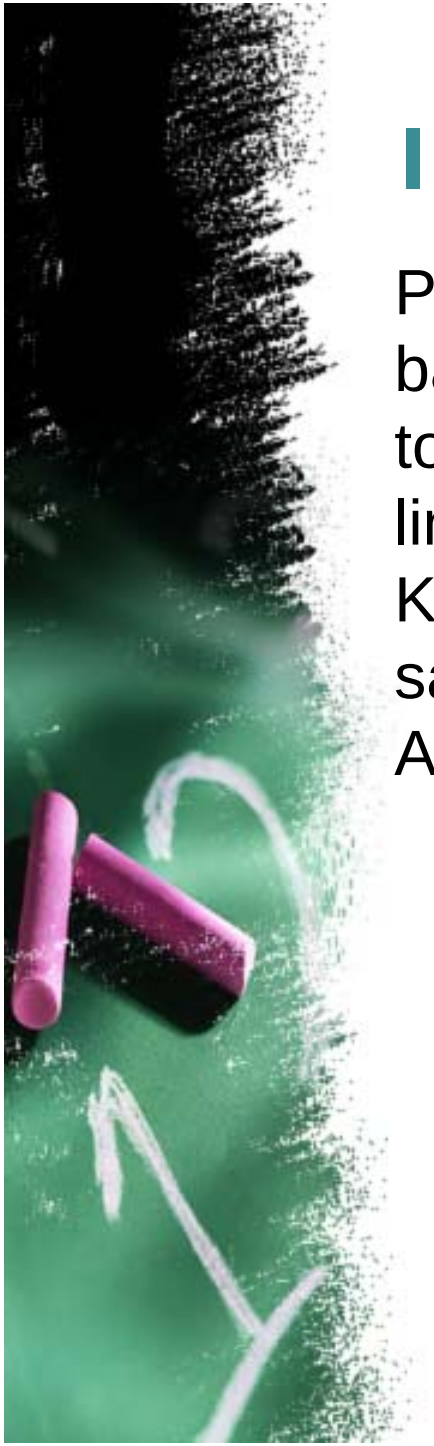
Solution:

Computing some powers of 7 we get:

$$7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, 7^5 = 16807, 7^6 = 117649.$$

Notice that the units digits have a pattern $7, 9, 3, 1, 7, 9, \ldots$ that is 4 terms long. We just need to determine at what point in the pattern we'll be with 2014. Dividing 2014 by 4 gives 503 with a remainder of 2. This means that we'll be at the 2nd term in the pattern and the units digit will be a **9**.

Runners: Please pass out Individual Question #3 face down.



Individual Question #3

Patricia leaves Grove City at 8:15 a.m. headed back to college after summer break. Since she is towing a trailer with all of her belongings, she is limited to an average speed of 45 mph. Her friend Kyle leaves Grove City two hours later taking the same route averaging the speed limit of 65 mph. At what time will Kyle pass Patricia?

Individual Question #3 - Clue

Patricia leaves Grove City at 8:15 a.m. headed back to college after summer break. Since she is towing a trailer with all of her belongings, she is limited to an average speed of 45 mph. Her friend Kyle leaves Grove City two hours later taking the same route averaging the speed limit of 65 mph. At what time will Kyle pass Patricia?

Clue: Remember the equation Distance = rate x time.

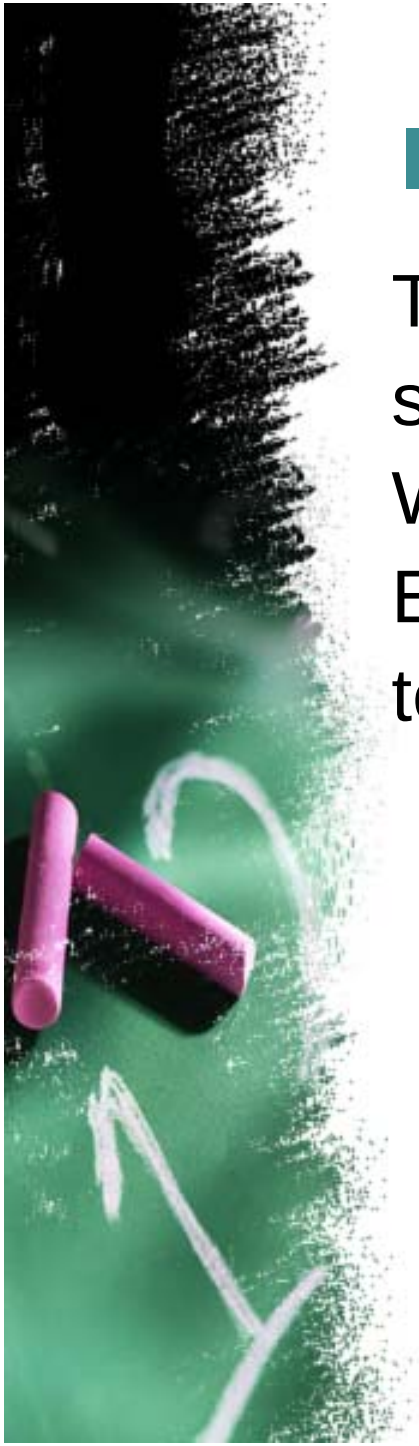
Individual Question #3 - Solution

Patricia leaves Grove City at 8:15 a.m. headed back to college after summer break. Since she is towing a trailer with all of her belongings, she is limited to an average speed of 45 mph. Her friend Kyle leaves Grove City two hours later taking the same route averaging the speed limit of 65 mph. At what time will Kyle pass Patricia?

Solution:

Let x be the amount of time that Patricia has been on the road in hours. Then $x - 2$ is the amount of time that Kyle has been on the road. Using the equation $Distance = rate \times time$ we know the distance that Patricia has gone is $45x$ and the distance that Kyle has gone is $65(x - 2)$. We want to solve for when these distances are equal so we have $45x = 65x - 130$ which gives $x = \frac{130}{20} = \frac{13}{2}$ hours. Then $x = 6$ hours and 30 minutes which means Kyle will pass Patricia at **2:45 p.m.**

Runners: Please pass out Individual Question #4 face down.



Individual Question #4

Two primes, not necessarily different, are selected from the first 9 prime numbers. What is the probability that their sum is odd? Express your answer as a fraction in lowest terms.

Individual Question #4 - Clue

Two primes, not necessarily different, are selected from the first 9 prime numbers. What is the probability that their sum is odd? Express your answer as a fraction in lowest terms.

Clue: 2 is the smallest prime number.

Individual Question #4 - Solution

Two primes, not necessarily different, are selected from the first 9 prime numbers. What is the probability that their sum is odd? Express your answer as a fraction in lowest terms.

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Solution:

The first 9 prime numbers are $2, 3, 5, 7, 11, 13, 17, 19$ and 23.

There are 9 possible values for the first of the two primes and 9 possible values for the second of the two primes for a total of $9 \times 9 = 81$ total pairs.

Since an even number plus an odd number is odd and an odd number plus an odd number is even, we want to count pairs with one even and one odd number. This is equal to the number of pairs that have a 2 as the first prime, which is 8, or two as the second prime, which is 8 also, giving us 16 pairs with an even sum. Thus the probability is $\mathbf{\frac{16}{81}}$.

Snack Time

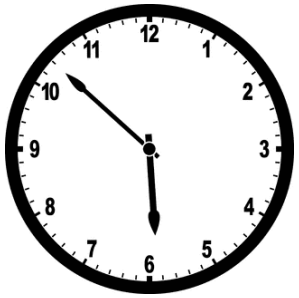
Runners: Please pass out Group Question #2 face down and the green Group Answer Sheet #2.

Group Question #2

How many degrees are in the smallest angle formed by the hour hand and the minute hand of a clock at 5:52?

Group Question #2 - Solution

How many degrees are in the smallest angle formed by the hour hand and the minute hand of a clock at 5:52?



Solution:

We'll solve this by seeing how many degrees each hand has rotated starting from 12.

The minute hand has rotated $\frac{52}{60}$ of the way around the clock which is a total of $\frac{52}{60} \times 360° = 312°$. The hour hand rotates $\frac{1}{12}$ of the way around the clock every hour which is $\frac{1}{12} \times 360° = 30°$. So at 5:52 the hour hand has rotated $5 \times 30 + \frac{52}{60} \times 30 = 176°$. Then the angle between them is $312° - 176° = 136°$.

Runners: Please pass out Individual Question #5 face down.

Individual Question #5

Find the whole number value of

$$\sqrt{1+3+5+7+9+11+\cdots+267+269+271+273+275}$$

Individual Question #5 - Clue

Find the whole number value of

$$\sqrt{1+3+5+7+9+11+\cdots+267+269+271+273+275}$$

Clue: $275+1=276$

Individual Question #5 - Solution

Find the whole number value of

$$\sqrt{1+3+5+7+9+11+\cdots+267+269+271+273+275}$$

Solution:

Summing the first and the last numbers in the square root gives $275+1=276$, summing second and the second to last number gives $3+273=276$, and this pattern of being able to pair up numbers in the sum to get 276 continues.

Since $\frac{275-1}{2}=137$ there are 138 odd numbers in the list which means that there are $\frac{138}{2}=69$ pairs that sum to 276.

Thus the total sum under the square root is $276\times 69=19044$ and the answer is $\sqrt{19044}=\mathbf{138}$.

Runners: Please pass out Individual Question #6 face down.

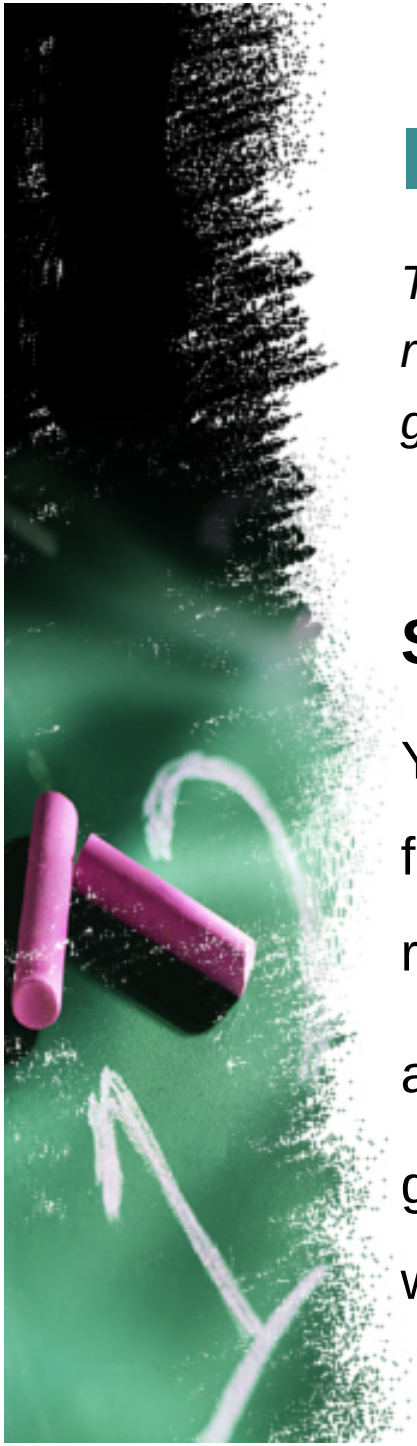
Individual Question #6

The Pirates lost 10 of their first 13 games. By winning 75% of their remaining games, they ended with victories in exactly $\frac{2}{3}$ of all their games. In all, how many games did they win?

Individual Question #6 - Clue

The Pirates lost 10 of their first 13 games. By winning 75% of their remaining games, they ended with victories in exactly $\frac{2}{3}$ of all their games. In all, how many games did they win?

Clue: The pirates played more than 65 games total.



Individual Question #6 - Solution

The Pirates lost 10 *of their first* 13 *games. By winning* 75% *of their remaining games, they ended with victories in exactly* $\frac{2}{3}$ *of all their games. In all, how many games did they win?*

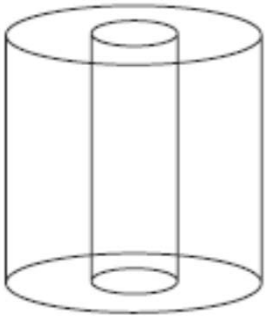
Solution:

You could use a guess and check approach here or do the following. Let x be the ratio factor. Then they won $3x$ of the remaining $4x$ games. In total the Pirates won $3 + 3x$ games and played $13 + 4x$ games. Thus $\frac{3+3x}{13+4x} = \frac{2}{3}$. Cross multiplying gives $9 + 9x = 26 + 8x$ which simplifies to $x = 17$. The Pirates won $3 + 3x = 3 + 3(17) =$ **54 games**.

Runners: Please pass out Individual Question #7 face down.

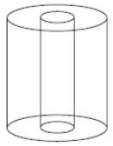
Individual Question #7

A hole of radius 3 inches is bored through the center of a right cylinder with radius 9 inches and height 10 inches. What is the total surface area of the resulting solid? Express your answer in terms of π.

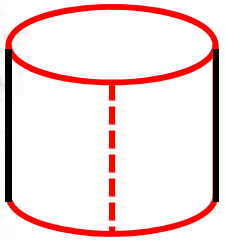


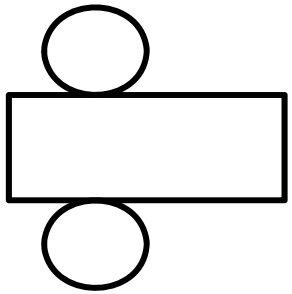
Individual Question #7 - Clue

A hole of radius 3 inches is bored through the center of a right cylinder with radius 9 inches and height 10 inches. What is the total surface area of the resulting solid? Express your answer in terms of π.



Clue: If you cut a cylinder along the red lines shown you get when you unfold it.





Individual Question #7 - Solution

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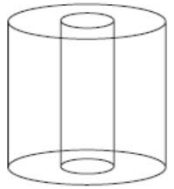
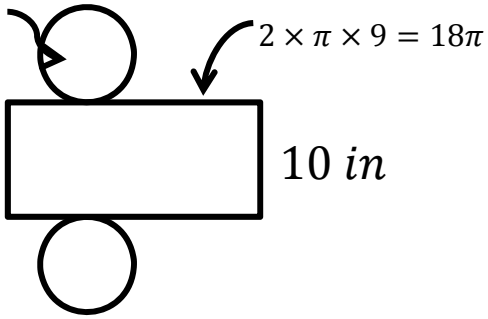


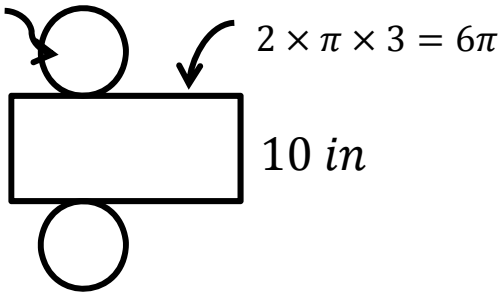
Solution:

$Area = \pi \times 9^2 = 81\pi$



Larger cylinder

$Area = \pi \times 3^2 = 9\pi$



Smaller cylinder

The surface area is the sum of the areas of the two rectangles and the area of the bigger circle minus the area of the smaller circle:

$$10(18\pi) + 10(6\pi) + 2(81\pi) - 2(9\pi) = \mathbf{384\pi}$$

Runners: Please pass out Individual Question #8 face down.

Individual Question #8

The first and seventh terms of a sequence are each 14. Starting with the third term, each term is the sum of the previous two terms. What is the fifth term?

Individual Question #8 - Clue

The first and seventh terms of a sequence are each 14. Starting with the third term, each term is the sum of the previous two terms. What is the fifth term?

Clue: The terms of the sequence can be negative.

Individual Question #8 - Solution

The first and seventh terms of a sequence are each 14*. Starting with the third term, each term is the sum of the previous two terms. What is the fifth term?*

Solution:

If the next term in the sequence were positive then each successive term would be greater than 14 so the next term has to be negative. We can use a guess and check approach to find the one that works:

$$14, -7, 7, 0, 7, 7, 14$$

Thus the 5^{th} term in the sequence is **7**.

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