Applied Optimization Examples

General steps:

1. Draw a picture and assign variables.
2. Write down the equation to be maximized or minimized (this is sometimes called the *objective equation*) and the equation that describes the constraint (this is sometimes called the *constraint equation*).
3. Use the constraint equation to rewrite the objective equation so that it has only one independent variable.
4. Find the domain of the [new] objective equation.
5. Find the maximum or minimum using calculus. Technically we're finding an absolute max/min, but in these problems it very often occurs at a local max/min in the domain (and in many problems there's only one possibility).
6. Verify your answer using either the First Derivative Test or the Second Derivative Test.
7. Be sure you answer the question that the problem is asking!

Exercises

1. A farmer wants to build a pen with two dividers in order to separate elephants, donkeys, and penguins. If 600 ft of fence is available and one side of the pen is bounded by a river and needs no fence since all the animals just happen to have an irrational fear of water, then what is the maximum area that can be enclosed?
2. A carpenter wants to build a rectangular box with square sides in which to put round things. The material for the bottom costs \$20/ft^2, material for the sides costs \$10/ft^2, and the material for the top costs \$50/ft^{2}t. If the volume of the box must be 5 ft^3, then find the dimensions that will minimize the cost (and find the minimum cost).
3. A knight sees a damsel in distress 3 miles downstream on the opposite side of a straight raging river 0.5 miles wide. The knight can swim at 4 mi/hr and run at 7 mi/hr. At what point on the opposite side should the knight swim in order to reach the distressed damsel as soon as possible.
4. A box with a square base and open top must have a volume of 32000 cm^3. Find the dimensions of the box that will minimize the amount of material needed.
5. A coffee shop has \$1000 in fixed daily costs and daily costs of \$60 per seat. The daily revenue per seat is \$90 if there are 100 or fewer seats. However, if the seating capacity is more than 100 places, the daily profit per seat will be decreased by \$1 for each additional seat over 100. If the fire marshal will only allow up to 150 seats, what should the seating capacity be in order to maximize the coffee shop's daily profit?
6. [Taken from the *Applied Calculus* book.] The total cost in dollars for Alicia to make q oven mitts is given by $C(q) = 64 + 1.5q + 0.01q^2$.
   (a) What is the fixed cost?
   (b) Find a function that gives the marginal cost.
   (c) Find a function that gives the average cost.
   (d) Find the quantity that minimizes the average cost.
   (e) Confirm that the average cost and marginal cost are equal at your answer to part 6d.

7. Solutions to the following three problems can be found here:
http://www2.gcc.edu/dept/math/faculty/BancroftED/teaching/handouts/ls_optimization_examples.pdf
To hear the embedded audio you must save the file to your computer and open it in Abobe Reader – the sound won't play if you open it in your web browser.

   (a) A 5 in $\times$ 8 in piece of paper has a square cut out of each corner (same size from each) and is then folded to make an open-top box. Find the size of the square that will maximize the volume.

   (b) Find the area of the largest rectangle that can be inscribed inside an isosceles triangle with side lengths $\sqrt{2}$, $\sqrt{2}$, 2.

   (c) A right circular cylinder is inscribed in a sphere of radius 6 in. Find the largest possible volume of such a cylinder.