

Introduction to *Mathematica*

⌘TeX file: *IntroMathematica-nb-all* — Daniel A. Graham <daniel.graham@duke.edu>, June 30, 2005

This is a brief introduction to those features of Mathematica that you will find most useful for this course. At Duke, see <http://www.oit.duke.edu/site/software/mathematica.html> for information about obtaining *Mathematica*.

Basics

I am preparing this document using the notebook interface for the Unix version of Mathematica so the keyboard shortcuts I describe may be a bit different if you're using either the Windows or Mac versions. Though *Mathematica* has both a notebook or graphics interface and a character-based interface, you will most likely be using the notebook interface and this differs little between operating systems. When you save your notebook you'll get a standard ascii notebook file that is exactly the same no matter which operating system you're using. You could email this file to a friend, for example, and he/she would be able to use it no matter which operating systems the two of you are using. You could also select File/SaveasSpecial/HTML or File/SaveasSpecial/TEX, as I will do with this document to display it on the internet.

When you start Mathematica, the screen will be blank save for a horizontal line across the top of the screen. This line represents the insertion point for a "cell". A cell can contain text, such as this paragraph, a headline, such as "Introduction to Mathematica" at the beginning of this document, input to be processed by Mathematica, output returned by Mathematica and so forth. The default is input but you can change this while the horizontal line is visible by selecting from the Format/Style. Once you start typing, the line will disappear and the characters you type will appear together with a "bracket" in the right-hand margin. You can click on this bracket and make another selection from the Format/Style menu if you change your mind about the style you want.

If you move the cursor to the end (or beginning) of a cell, a new insertion line will appear where you can, once again, select a style and enter new material. You can also, of course, return to any existing cell and make any changes you like.

Input Expressions

Simply enter an expression, say $(2+3)^3$, in an input cell and press the shift key and the enter key at the same time. *Mathematica* will process your input, label your input cell with "In[#]" and return its output in a cell labeled "Out[#]" where "#" is the number Mathematica assigns to this matched pair of cells.

In: $(2 + 3) ^ 3$

Out: 125

If you edit your input cell, "In[#]" will disappear to remind you that your new input has not yet been processed.

The operators are what you would expect, with * or space for multiplication and ^ for exponentiation. Watch out for the following common sources of consternation:

Spaces

$3x$ will be interpreted as 3 times x and not as a variable named $3x$. On the other hand, $x3$ will be interpreted as a variable named $x3$ and not as the product of x times 3. If what you want is x times 3 and you want to put the x first, then you must put a $*$ or a space between the x and the 3.

Cases

Mathematica is case sensitive, e.g., y and Y are the names of two different variables. *Mathematica's* built in variables, like Pi , and functions, like Log , always have at least the first letter capitalized and are usually not abbreviated but are spelled out in full. You can avoid conflicts by not capitalizing the first letter of your own variables and functions.

Delimiters

Mathematica uses the delimiters $()$, $[]$, $\{ \}$ and $[[]]$ for completely different purposes. Parentheses are used only for grouping, e.g., $(x+y)^2$. Square brackets are used only to provide the argument(s) to a function, e.g., $\text{Log}[5]$. Curly braces are used only to delimit the elements of a list, e.g., $\text{mylist}=\{x,y\}$ defines mylist to be a list containing two elements - the variables x and y . Double square brackets are used only to refer to elements of a list, e.g., $\text{mylist}[[2]]$ refers to the second element of mylist - the variable y .

Symbols and Numbers

You are not limited to numeric entries, *Mathematica* will just as happily process symbolic expressions.

In: $(x + y)^3$

Out: $(x + y)^3$

In: $\text{Expand}[(x + y)^3]$

Out: $x^3 + 3x^2y + 3xy^2 + y^3$

When processing numbers, *Mathematica's* default is to treat them as exact:

In: $1/3 + 2/5 + 3/7$

Out: $\frac{122}{105}$

If you want a decimal approximation you can either ask for it explicitly using the built-in numerical value function, N :

In: $N[1/3 + 2/5 + 3/7]$

Out: 1.1619

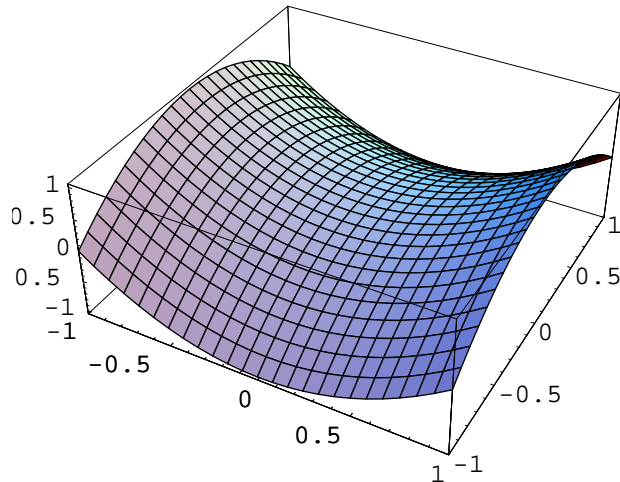
or you can enter a decimal in your input:

In: $1/3 + .4 + 3/7$

Out: 1.1619

Mathematica can compute very large numbers

In: 200!



Out: –SurfaceGraphics–

where both x and y go from -1 to +1.

By the way, if you forget how a function works, just put the cursor after the name of the function, e.g., after the Plot3D, click on Help and then Find Selected Function and the complete documentation on the relevant function will pop up. Mathematica's on-line help system is the best I've seen.

Using Prior Expressions

Using "%" to refer to the previous expression

The percent sign, %, is shorthand for the results of the previous calculation, e.g.,

In: Expand [(x + y) ^ 3]

Out: $x^3 + 3x^2y + 3xy^2 + y^3$

In: Factor [%]

Out: $(x + y)^3$

Using "=" to name an expression

Alternatively, you can name an expression using "="

In: myexpression = $x^2 - 8x + 15$

Out: $15 - 8x + x^2$

and later refer to the equation by name

In: mysolutions = Solve [myexpression == 0]

Out: $\{\{x \rightarrow 3\}, \{x \rightarrow 5\}\}$

Note the use of the single equality to name or define an expression and the use of the double equality for "equals". Note also that the output of "Solve" is a list of the two solutions to the equation. To refer, say, to the second solution you would use mysolutions[[2]].

Mathematica never forgets. If you ever enter $x = 3$, even by mistake, then the variable named x will thereafter be replaced by the number 3. Deleting the cell containing the definition won't help. To remove this definition from memory you need to enter Clear[x].

Using "/" to substitute values into an expression

You can substitute values into an expression using the substitution operator "/", e.g.,

In: $x^2/.x \rightarrow 3$

Out: 9

where the rightharrow is gotten by typing "-" and then ">", or

In: $(a + b)^2/.a + b \rightarrow x - y$

Out: $(x - y)^2$

or

In: `myexpression/.mysolutions`

Out: {0,0}

In the last case, the list of "mysolutions", namely $x \rightarrow 3$ and $x \rightarrow 5$ are substituted one at a time into "myexpression". The fact that the result in each case is zero, confirms that 3 and 5 are both solutions to $\text{myexpression} = 0$.

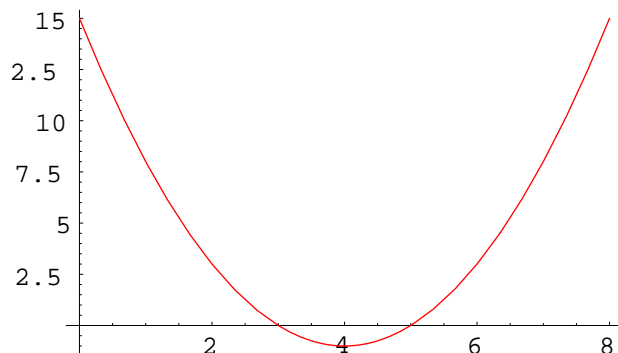
Using ":=" to define a function

You can define your own functions using ":=".

In: `f[z_]:=z^2 - 8z + 15`

Note the use of "z_" on the left to refer to "z" on the right though you could equally well use "a_" and "a" or any other such pair of symbols. Now f can be used exactly as you would any other function, e.g., to confirm visually "mysolutions".

In: `Plot[f[x], {x, 0, 8}, PlotStyle -> RGBColor[1, 0, 0]]`



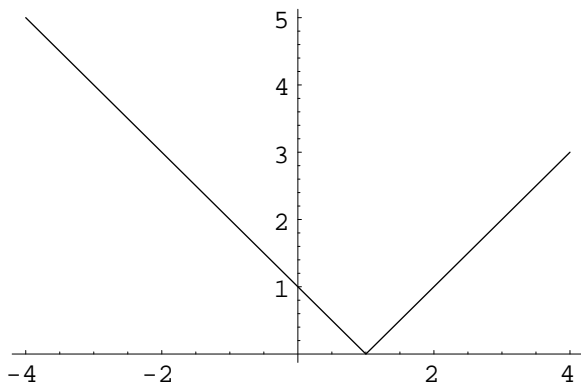
Out: -Graphics-

You can also define piecewise fundtions

In: `g[x_]:=1 - x/; x <= 1`

`g[x_]:=x - 1/; x > 1`

In: `Plot[g[x], {x, -4, 4}]`



Out: –Graphics–

Using "Simplify" to simplify an expression

In: $\text{mess} = (3 + 7x + 8x^2 + 5x^3 + x^4) / (3 + 10x + 18x^2 + 14x^3 + 3x^4)$

Out: $\frac{3 + 7x + 8x^2 + 5x^3 + x^4}{3 + 10x + 18x^2 + 14x^3 + 3x^4}$

In: Simplify[mess]

Out: $\frac{1 + x + x^2}{1 + 2x + 3x^2}$

Forgetting

Mathematica never forgets. If you ever enter $x=3$, even by mistake, then the variable named x will thereafter be replaced by the number 3. Deleting the cell containing the definition won't help. To remove the definition from memory you need to enter `Clear[x]`:

In: $x = 3$

Out: 3

In: x

Out: 3

In: `Clear[x]`

In: x

Out: x

You can remove all of your own definitions either with the following magic:

In: `Clear["Global`*"]`

In: myexpression

Out: myexpression

or by choosing "Kernel / Quit Kernel / Local" from the drop-down menu. You can then make any changes you like and choose "Kernel / Evaluation / Evaluate Notebook" from the drop-down menu to redo all the calculations in your notebook.

Commonly Used Commands

Using "D" to differentiate

Taking the derivative of an expression with respect to a variable is such a common operation that it is one of the few Mathematica commands that is abbreviated. Here are some examples.

In: $D[3x^2, x]$

Out: $6x$

In: $D[f[x], x]$

Out: $f'[x]$

The partial derivative of a function of x and y with respect to y

In: $D[3x^2 + 2xy + 2y^3, y]$

Out: $2x + 6y^2$

Note: In the expression "2 times x times y ", there's a space between the x and the y , otherwise the expression would be interpreted as 2 times the variable named xy .

The generalization of the "product rule" to three functions

In: $D[a[x]b[x]c[x], x]$

Out: $b[x]c[x]a'[x] + a[x]c[x]b'[x] + a[x]b[x]c'[x]$

The generalization of the "chain rule" to three functions

In: $D[a[b[c[x]]], x]$

Out: $a'[b[c[x]]]b'[c[x]]c'[x]$

Using "Integrate"

One that even I could do without much effort

In: $\text{Integrate}[x^3, x]$

Out: $\frac{x^4}{4}$

and one that I couldn't

In: $\text{Integrate}[\text{Sin}[x]^2, x]$

Out: $\frac{x}{2} - \frac{1}{4}\text{Sin}[2x]$

Is the last one correct?

In: $D[\%, x]$

Out: $\frac{1}{2} - \frac{1}{2}\text{Cos}[2x]$

In: $\text{Simplify}[\%]$

Out: $\text{Sin}[x]^2$

Yes. Note that for indefinite integrals, *Mathematica* does not display the constant of integration. For a definite integral give the range of integration

In: $\text{Integrate}[x^3, \{x, 0, 1\}]$

Out: $\frac{1}{4}$

Using "Solve"

To solve a simple equation:

In: Solve [$x^{(2/3)} == 2x$]//TableForm

Out: $x \rightarrow 0$
 $x \rightarrow \frac{1}{8}$

or a more complicated one:

In: $e = x^3 - 2x + 9$;

In: Solve [$e == 0$]//TableForm

$x \rightarrow -2 \left(\frac{2}{3(81 - \sqrt{6465})} \right)^{1/3} - \frac{\left(\frac{1}{2}(81 - \sqrt{6465}) \right)^{1/3}}{3^{2/3}}$
Out: $x \rightarrow (1 + i\sqrt{3}) \left(\frac{2}{3(81 - \sqrt{6465})} \right)^{1/3} + \frac{(1 - i\sqrt{3}) \left(\frac{1}{2}(81 - \sqrt{6465}) \right)^{1/3}}{23^{2/3}}$
 $x \rightarrow (1 - i\sqrt{3}) \left(\frac{2}{3(81 - \sqrt{6465})} \right)^{1/3} + \frac{(1 + i\sqrt{3}) \left(\frac{1}{2}(81 - \sqrt{6465}) \right)^{1/3}}{23^{2/3}}$

Note that we get one real solution, the first, and two complex solutions. Let's see whether the first solution works:

In: $e / \% [[1]]$

Out: $9 - 2 \left(-2 \left(\frac{2}{3(81 - \sqrt{6465})} \right)^{1/3} - \frac{\left(\frac{1}{2}(81 - \sqrt{6465}) \right)^{1/3}}{3^{2/3}} \right) + \left(-2 \left(\frac{2}{3(81 - \sqrt{6465})} \right)^{1/3} - \frac{\left(\frac{1}{2}(81 - \sqrt{6465}) \right)^{1/3}}{3^{2/3}} \right)^3$

Is this really zero?

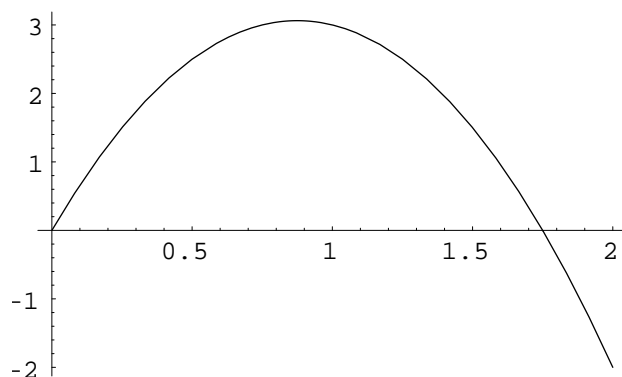
In: Simplify [%]

Out: 0

To find the maximum of a function of a single variable:

In: payoff = $7x - 4x^2$;

In: Plot [payoff, {x, 0, 2}]



Out: -Graphics-

In: Solve [D [payoff, x] == 0]

Out: $\left\{ \left\{ x \rightarrow \frac{7}{8} \right\} \right\}$

In: payoff/.[[1]]

Out: $\frac{49}{16}$

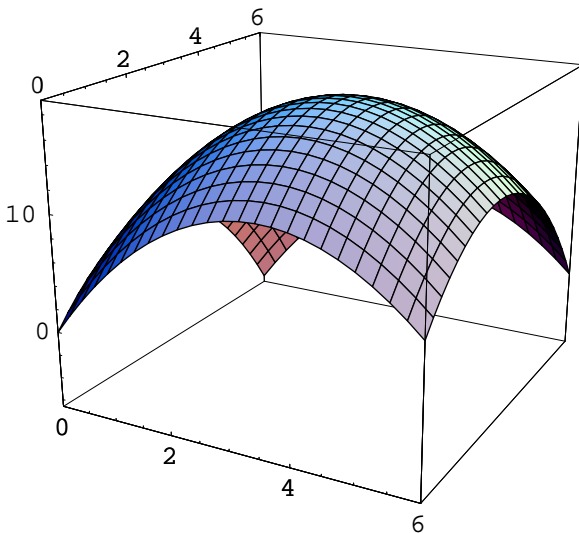
In: %//N

Out: 3.0625

To find the maximum of a function of two variables

In: payoff = 7x + 5y - x² - y²;

In: Plot3D [payoff, {x, 0, 6}, {y, 0, 6}, ViewPoint-> {1.5, -2.8, 1.1}, BoxRatios -> {1, 1, .8}]



Out: -SurfaceGraphics-

Note that you can interactively set the "ViewPoint" by putting your cursor at that point in the expression, selecting "Input / 3D View Point Selector" from the drop-down menu and the clicking on "Paste" when you're happy with the result.

In: Solve [{D [payoff, x] == 0, D [payoff, y] == 0}]/TableForm

Out: $x \rightarrow \frac{7}{2} \quad y \rightarrow \frac{5}{2}$

↪ Converted by m2l 1.11 (2005/06/27)