Newton's Law of Cooling (and Heating)

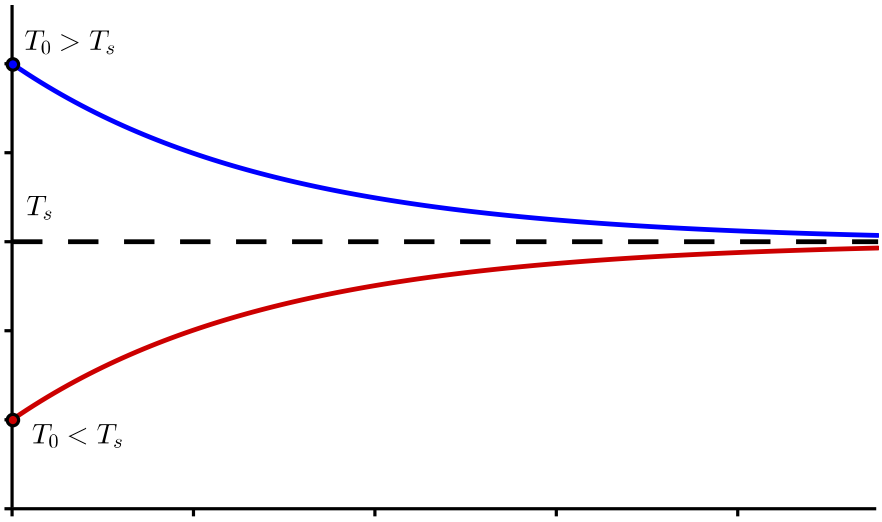
Formula

Let T_0 be the initial temperature of an object and let T_s be the temperature of the environment surrounding the object (T_s is assumed to be constant). Then then temperature of the object at time t is given by

$$T(t) = T_s + (T_0 - T_s)e^{kt}$$

where k is a constant of proportionality which gives the rate at which the object heats or cools.

The figure below shows the general shape of $T(t)$ when $T_0 > T_s$ (i.e., in a cooling scenario) and when $T_0 < T_s$ (i.e., in a heating scenario):



Examples and Homework Problems

1. Mr. Body has been found dead at 8:05 a.m. in the _____ (where the temperature is 70°F), murdered by _____ with the _____. His temperature at that time is found to be 94°F. Three hours later, the police allow Mr. Body's body to be taken away by the coroner, who finds Mr. Body's temperature to be 81°F. Medical records indicate that Mr. Body's normal body temperature is 99°F. Where and when did Mr. Body die? Who killed him and which weapon was used?

2. A mug of hot water whose initial temperature is _____°F is placed in a _____°F room. After 10 minutes the temperature of the water is _____°F. What will the temperature of the water be at the end of class (i.e., after _____ minutes)?

3. Suppose we are preparing a lovely *Canard à l'Orange* (roast duck with orange sauce). We first take our duck out of a 36°F refrigerator and place it in a 350°F oven to roast. After 10 minutes the internal temperature is 53°F. If we want to roast the duck until just under well-done (about 170°F internally), when will it be ready?

4. A cake is removed from a 180°F cake is removed from an oven and placed in a 70°F room. 3 minutes later it has cooled to 160°F.

   (a) What is its temperature be after 20 minutes? (*98.87°F*)

   (b) How long will it take for the cake to cool to 90°F? (*25.49 minutes*)

5. A small metal bar whose temperature is 30°C is dropped into a container of 75°C water. After 1 second the temperature of the bar has increased 1°C.

   (a) How long will it take for the temperature of the bar to reach 70°C? (*97.77 seconds*)

   (b) 74°C? (*169.39 seconds*)

6. A 40°F roast is cooked in a 350°F oven. After 2 hours, the temperature of the roast is 125°F. The roast is done when the internal temperature reaches 165°F. When will the roast be done? (*3.22 hours after going in the oven.*)

7. **(Challenge Problem)** A thermometer is taken from an inside room to the outdoors, where the air temperature is 5°F. After 1 minute the thermometer reads 55°F, and after 5 minutes it reads 30°F. What was the temperature of the room? (*64.46°F*)