Quadric Surface Examples

Example 1 (12.6.30).

\[ 4x^2 - y + 2z^2 = 0 \iff y = \frac{x^2}{1/4} + \frac{z^2}{1/2} \]

This is a paraboloid with vertex at the origin, opening along the y-axis.

![Figure 1: Paraboloid](image)

Example 2 (12.6.36).

\[ x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0 \iff \frac{(x - 1)^2}{2} - \frac{(y - 1)^2}{2} + \frac{(z + 2)^2}{2} = 1 \]

This is a hyperboloid of one sheet centered at \((1, 1, -2)\).
Example 3 (12.6.15).

\[-x^2 + 4y^2 - z^2 = 4 \iff\]

This is a hyperboloid of two sheets centered at the origin.

![Hyperboloid of two sheets](image)

Figure 3: Hyperboloid of two sheets

Example 4 (12.rev.35).

\[4x^2 + 4y^2 - 8y + z^2 = 0 \iff\]

This is an ellipsoid centered at \((0, 1, 0)\).

![Ellipsoid](image)

Figure 4: Ellipsoid
**Example 5** (12.rev.31).

\[ x^2 = y^2 + 4z^2 \quad \Leftrightarrow \]

This is a cone with vertex at the origin and axis parallel to the \( x \)-axis.

![Figure 5: Cone](image)

**Example 6** (12.6.20).

\[ x = y^2 - z^2 \quad \text{vs.} \quad y = z^2 - x^2 \]

This are both hyperbolic paraboloids, one first with the \( x \)-axis perpendicular to the ‘saddle’ and the other with the \( y \)-axis perpendicular to the ‘saddle’.

![Figure 6: Hyperbolic Paraboloids](image)